

Strong decays of higher excited heavy-light mesons in a chiral quark model

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The strong decay properties of the higher excited heavy-light mesons from the first radially excited states up to the first F -wave states are studied in a constituent quark model. It is found that many missing excitations have good potentials to be found in future experiments for their narrow widths, and some of them dominantly decay into the first orbital excitations rather than into ground states. In future observations, one should focus on the decay processes not only into the ground states, but also into the low-lying P -wave excitations with $J^P = 0^+, 1^+$. Furthermore, the nature of the newly observed states $D_J(3000)$, $D_J^*(3000)$ and $B(5970)$ is discussed. It is predicted that $D_J(3000)$ seems to be a partner of $D_{sJ}(3040)$, which could be identified as the low-mass mixed state $|2P_1\rangle_L$ ($J^P = 1^+$) via the 2^1P_1 - 2^3P_1 mixing. The $D_J^*(3000)$ resonance seems to favor the 1^3F_4 state, while the quantum numbers $J^P = 0^+$ and 2^+ cannot be excluded completely, more experimental observations are needed to determine its J^P values. The $B(5970)$ resonance is most likely to be the 1^3D_3 with $J^P = 3^-$.

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I. INTRODUCTION

During the past several years, significant progress has been made in the observation of the heavy-light mesons. More and more higher excitations have been found in experiments. In the D -meson family, four new members, the $D(2550)$, $D(2600)$, $D(2750)$ and $D(2760)$ were discovered by the *BABAR* Collaboration [1], which were confirmed by the LHCb Collaboration with slightly different masses [2]. Furthermore, the LHCb Collaboration observed two new higher D -meson excitations, $D_J^*(3000)$ and $D_J(3000)$, with natural and unnatural parities, respectively [2]. In the D_s -meson family, except for the $D_{s1}(2700)$ and $D_{sJ}(2860)$ observed by *BABAR* and Belle before [3, 4], recently a new broad higher D_s -meson excitation $D_{sJ}(3040)$ was observed by *BABAR* as well [5]. Very recently, progress has been obtained in the search for the excited B meson as well. The CDF Collaboration observed some evidence of a new narrow excited state $B(5970)$ in the $B\pi$ channel [6].

About the newly observed states $D(2550)$, $D(2600)$, $D(2750)$, $D(2760)$, $D_{s1}(2700)$, $D_{sJ}(2860)$ and $D_{sJ}(3040)$, their strong decays had been analyzed in our previous papers [7–9]. We found that the $D(2760)$ could be identified as the 1^3D_3 with $J^P = 3^-$, while the $D(2750)$ is most likely to be the mixed state $|1D_2'\rangle_H$ ($J^P = 2^-$) via the 1^1D_2 - 1^3D_2 mixing. The $D(2600)$ favors the mixed state $|(SD)_1\rangle_L$ ($J^P = 1^-$) via the 1^3D_1 - 2^3S_1 mixing. The $D(2550)$ might be assigned as the 2^1S_0 assignment although its predicted width is obviously narrower than that given in experiments. For the newly observed D_{sJ} mesons, we found that the $D_{s1}(2700)$ is most likely to be the strange partner of the $D(2600)$. There might exist two-state scenario for the $D_{sJ}(2860)$, one resonance corresponds to the strange partner of the $D(2760)$ [denoted by $D_{sJ_1}(2860)$] and the other resonance is the strange partner of the $D(2750)$ [denoted by $D_{sJ_2}(2860)$]. The $D_{sJ}(3040)$ could be identified as the low-mass physical state $|2P_1\rangle_L$ via the

2^1P_1 - 2^3P_1 mixing. More discussions of these newly observed D and D_s mesons can be found in the literature [10–30].

On the other hand, about the higher excitations $D_J^*(3000)$, $D_J(3000)$ and $B(5970)$ observed very recently, a few studies can be found in the literature [31–37]. For example, the $D_J^*(3000)$ and $D_J(3000)$ states were explained as the first P -wave states in the D meson family with $J^P = 0^+$ and 1^+ , respectively [23]. As a candidate of the 2^3S_1 [36, 37], 1^3D_1 and 1^3D_3 states [36], the strong decay properties of the $B(5970)$ were studied with the heavy meson effective theory.

These newly observed higher excitations provide us a good chance to establish a fairly abundant D - and D_s -meson spectroscopy, which has been summarized in Tab. I. For comparisons, some interested predictions of the heavy-light meson spectroscopy in theory are also included [39, 40]. From the table, it is seen that the knowledge about the B - and B_s -meson spectroscopy is very poor. Except for the ground states, only two low-lying orbital excitations in the P waves are found in the B - and B_s -meson spectroscopy. The discovery of the $B(5970)$ enhances our confidence in the search for more higher excitations in the B - and B_s -meson families. From now on, we might have a golden time to study the higher excitations of the heavy-light mesons.

To provide helpful information for the experimental search for more excited heavy-light mesons and to gain a unified understanding of the newly observed resonances, in this work we continue to study the strong decay properties of the higher excitations of heavy-light mesons up to the F waves (see Tab. I) with the chiral quark model as well. This model has been developed and successfully used to deal with the strong decays of heavy-light mesons, charmed and strange baryons [7–9, 41–43].

This paper is organized as follows. In the subsequent section, a brief review of the model is given. The numerical results are presented and discussed in Sec. III. Finally, a summary is given in Sec. IV.

TABLE I: The heavy-light meson spectroscopy predicted in theory compared with the observations in experiments. The predicted masses (MeV) are obtained from [39, 40], while the observed states are obtained from the PDG [38]. The $1P_1$ and $1P'_1$ stand for the low-mass and high-mass mixed states via the 1^1P_1 - 1^3P_1 mixing, respectively, which have been defined in [7]. The $1D_2$ and $1D'_2$ stand for the low-mass and high-mass mixed states via the 1^1D_2 - 1^3D_2 mixing, respectively. The $2P_1$ and $2P'_1$ stand for the low-mass and high-mass mixed states via the 2^1P_1 - 2^3P_1 mixing, respectively.

States	<u>D mesons</u>		<u>D_s mesons</u>		<u>B mesons</u>		<u>B_s mesons</u>	
$n^{2S+1}L_J$	Predicted Mass	Observed state	Predicted Mass	Observed state	Predicted Mass	Observed state	Predicted Mass	Observed state
1^1S_0	1868/1871	$D(1865)$	1965/1969	$D_s(1969)$	5279/5280	$B(5280)$	5373/5372	$B_s(5367)$
1^3S_1	2005/2010	$D^*(2007)$	2113/2111	$D_s^*(2112)$	5324/5326	$B^*(5325)$	5421/5414	$B_s^*(5415)$
1^3P_0	2377/2406	$D_0^*(2400)$	2487/2509	$D_{s0}(2317)$	5706/5749	?	5804/5833	?
$1P_1$	2417/2426	$D_1(2430)$	2535/2536	$D_{s1}(2460)$	5700/5723	$B_J^*(5732)?$	5805/5831	?
$1P'_1$	2490/2469	$D_1(2420)$	2605/2574	$D_{s1}(2536)$	5742/5774	$B_1(5721)$	5842/5865	$B_{s1}(5830)$
1^3P_2	2460/2460	$D_2(2460)$	2581/2571	$D_{s2}(2573)$	5714/5741	$B_2(5747)$	5820/5842	$B_{s2}(5840)$
2^1S_0	2589/2581	$D(2550)?$	2700/2688	?	5886/5890	?	5985/5976	?
2^3S_1	2692/2632	$D(2600)?$	2806/2731	$D_{s1}(2700)?$	5920/5906	?	6019/5992	?
1^3D_1	2795/2788	?	2913/2913	?	6025/6119	?	6127/6209	?
$1D_2$	2775/2806	?	2900/2931	?	5985/6102	?	6095/6189	?
$1D'_2$	2883/2850	$D(2750)?$	2953/2961	$D_{sJ_2}(2860)?$	6037/6121	?	6140/6218	?
1^3D_3	2799/2863	$D(2760)?$	2925/2971	$D_{sJ_1}(2860)?$	5993/6091	$B(5970)?$	6103/6191	?
2^3P_0	2949/2919	?	3067/3054	?	6163/6221	?	6264/6318	?
$2P_1$	2995/2932	$D_J(3000)?$	3114/3067	$D_{sJ}(3040)?$	6175/6209	?	6278/6321	?
$2P'_1$	3045/3021	?	3165/3154	?	6194/6281	?	6296/6345	?
2^3P_2	3035/3012	?	3157/3142	?	6188/6260	?	6292/6359	?
1^3F_2	3101/3090	?	3224/3230	?	6264/6412	?	6369/6501	?
$1F_3$	3074/3129	?	3203/3254	?	6220/6391	?	6332/6468	?
$1F'_3$	3123/3145	?	3247/3266	?	6271/6420	?	6376/6515	?
1^3F_4	3091/3187	?	3220/3300	?	6226/6380	?	6337/6475	?

II. THE MODEL

In this section, we give a brief review of the chiral quark model. The details of this model can be found in our previous papers [7–9]. In this model, the low energy quark-pseudoscalar-meson and quark-vector-meson interactions in the SU(3) flavor basis might be described by the effective Lagrangians

$$\mathcal{L}_{Pqq} = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \partial^\mu \phi_m, \quad (1)$$

$$\mathcal{L}_{Vqq} = \sum_j \bar{\psi}_j (a \gamma_\mu^j + \frac{ib}{2m_j} \sigma_{\mu\nu} q^\nu) V^\mu \psi_j, \quad (2)$$

respectively, where ψ_j represents the j th quark field in the hadron, ϕ_m is the pseudoscalar meson field, f_m is the pseudoscalar meson decay constant, and V^μ represents the vector meson field. Parameters a and b denote the vector and tensor coupling strength, respectively.

In this model, the wave function of a heavy-light meson is adopted by the nonrelativistic harmonic oscillator wave function, i.e., $\psi_{lm}^n = R_{nl} Y_{lm}$. To match the nonrelativistic wave functions of the heavy-light mesons, we should adopt the nonrelativistic form of Eq. (1) in the calculations, which is given

by [46–48]

$$H_m = \sum_j \left[A \sigma_j \cdot \mathbf{q} + \frac{\omega_m}{2\mu_q} \sigma_j \cdot \mathbf{p}_j \right] I_j \varphi_m, \quad (3)$$

in the center-of-mass system of the initial meson, where we have defined $A \equiv -(1 + \frac{\omega_m}{E_f + M_f})$. On the other hand, from Eq. (2), one can easily derive the nonrelativistic transition operators for the emission of a transversely or longitudinally polarized vector meson, which are given by [49–51]

$$H_m^T = \sum_j \left\{ i \frac{b'}{2m_q} \sigma_j \cdot (\mathbf{q} \times \epsilon) + \frac{a}{2\mu_q} \mathbf{p}_j \cdot \epsilon \right\} I_j \varphi_m, \quad (4)$$

and

$$H_m^L = \sum_j \frac{a M_v}{|\mathbf{q}|} I_j \varphi_m. \quad (5)$$

In the above equations, \mathbf{q} and ω_m are the three-vector momentum and energy of the final-state light meson, respectively; \mathbf{p}_j is the internal momentum operator of the j th quark in the heavy-light meson rest frame; σ_j is the spin operator corresponding to the j th quark of the heavy-light system; and μ_q is a reduced mass given by $1/\mu_q = 1/m_j + 1/m'_j$ with m_j and m'_j for the masses of the j th quark in the initial and final mesons,

respectively. M_v is the mass of the emitted vector meson. The plane wave part of the emitted light meson is $\varphi_m = e^{-i\mathbf{q}\cdot\mathbf{r}_j}$, and I_j is the flavor operator defined for the transitions in the SU(3) flavor space [47–51]. The parameter b' in Eq. (4) is defined as $b' \equiv b - a$.

For a light pseudoscalar meson emission in heavy-light meson strong decays, the partial decay width can be calculated with [7, 41]

$$\Gamma = \left(\frac{\delta}{f_m}\right)^2 \frac{(E_f + M_f)|\mathbf{q}|}{4\pi M_i(2J_i + 1)} \sum_{J_{iz}, J_{fz}} |\mathcal{M}_{J_{iz}, J_{fz}}|^2, \quad (6)$$

where $\mathcal{M}_{J_{iz}, J_{fz}}$ is the transition amplitude, and J_{iz} and J_{fz} stand for the third components of the total angular momenta of the initial and final heavy-light mesons, respectively. δ as a global parameter accounts for the strength of the quark-meson couplings. It has been determined in our previous study of the strong decays of the charmed baryons and heavy-light mesons [7, 41]. Here, we take the same value as that determined in Refs. [7, 41], i.e., $\delta = 0.557$.

In the calculation, the standard quark model parameters are adopted. Namely, we set $m_u = m_d = 330$ MeV, $m_s = 450$ MeV, $m_c = 1700$ MeV, and $m_b = 5100$ MeV for the constituent quark masses. The harmonic oscillator parameter β in the wave functions of heavy-light mesons is taken as $\beta = 0.40$ GeV. The decay constants for π , K and η mesons are taken as $f_\pi = 132$ MeV, $f_K = f_\eta = 160$ MeV, respectively. For the quark-vector-meson coupling strength which still suffers relatively large uncertainties, we adopt the values extracted from vector meson photoproduction, i.e. $a \simeq -3$ and $b' \simeq 5$ [49–51]. The masses of the mesons used in the calculations are adopted from the PDG [38]. With these parameters, the strong decay properties of the well-known heavy-light mesons and charmed baryons have been described reasonably [7–9, 41, 42].

III. CALCULATIONS AND RESULTS

A. The 2^1S_0 states

Recently, the BABAR Collaboration observed an excited D -meson state $D(2550)$ in the $D^*\pi$ channel with a width of $\Gamma \simeq 130$ MeV [1], which was also observed by the LHCb Collaboration recently [2]. In theory, the first radially excited D -meson state 2^1S_0 has a mass ~ 2.58 GeV [39, 40, 44]. Furthermore, the decay mode and the BABAR analysis of angle distributions indicate that the $D(2550)$ should be classified as the radially excited state 2^1S_0 . We have studied the strong decays of the $D(2550)$ as the first radially excited D -meson state 2^1S_0 in Refs. [7, 9]. The strong decays of $D(2^1S_0)$ are dominated by the $D^*\pi$ and $D_0(2400)\pi$ modes. In the present work we have improved our predictions according to the new observations from the LHCb Collaboration [2]. The results are listed in Tab. II. The predicted partial decay width ratio is

$$\frac{\Gamma[D^*\pi]}{\Gamma[D_0(2400)\pi]} \simeq 0.28, \quad (7)$$

which strongly depends on the mass of $D_0(2400)$. Our predicted width, $\Gamma \simeq 68$ MeV, is about a factor of 2 narrower than the data. The 3P_0 model [23] and relativistic quark model [39] calculations also obtained a narrow width for the $D(2^1S_0)$ state.

If the $D(2550)$ is the 2^1S_0 assignment indeed, the same excitation in the D_s -, B -, and B_s -meson spectroscopy might be found in future experiments. To obtain more information about these missing states, we further study the strong decay properties of these radially excited states $D_s(2^1S_0)$, $B(2^1S_0)$ and $B_s(2^1S_0)$ in the following.

TABLE II: The strong decay width (MeV) of the $D(2550)$ as the 2^1S_0 with a mass of 2580 MeV.

Mode	$D^*\pi$	$D_0(2400)\pi$	Total
Width	15	53	68

In the D_s -meson family, the predicted mass of the first radially excited state 2^1S_0 is around 2.7 GeV [39, 40, 44]. There is no experimental information about this state. The strong decay properties of $D_s(2^1S_0)$ have been studied in our previous work [7]. Its strong decays are governed by the D^*K channel. Based on our model, the $D_s(2^1S_0)$ is a very narrow state with a width of $\Gamma \simeq 10$ MeV. Such a narrow state should be observed in the D^*K final states if it indeed exists.

In the B -meson family, the predicted mass of $B(2^1S_0)$ is around 5.9 GeV [39, 40, 44]. We calculate its strong decay properties, which are listed in Tab. III. It is seen that the strong decays are dominated by the $B^*\pi$ and $B(1^3P_0)\pi$ channels, and the predicted partial width ratio is

$$\frac{\Gamma[B^*\pi]}{\Gamma[B(1^3P_0)\pi]} \simeq 1.3. \quad (8)$$

The $B(2^1S_0)$ is also a narrow state with a width of $\Gamma \simeq 40$ MeV, which is slightly narrower than that of $D(2^1S_0)$. The decay width predicted by us is roughly compatible with the recent calculations with 3P_0 model [33].

TABLE III: The strong decay width (MeV) of the $B(2^1S_0)$ with a mass of 5886 MeV. The mass of $B(1^3P_0)$ is taken as 5706 MeV.

Mode	$B^*\pi$	$B(1^3P_0)\pi$	$B(1^3P_2)\pi$	$B^*\eta$	Total
Width	16	24	0.01	0.25	40

In the B_s -meson family, the predicted mass of $B_s(2^1S_0)$ is around 6.0 GeV [39, 40, 44]. We calculate the strong decay properties of this state with a mass of $M = 5985$ MeV. This state dominantly decays into the B^*K channel. The total decay width is $\Gamma \simeq 26$ MeV, which is a little narrower than that of the 3P_0 calculations [33].

As a whole, the first radially excited states 2^1S_0 in the D -, D_s -, B - and B_s -meson spectroscopy might have a fairly narrow width. However, up to now none of them has been established. It is still a puzzle why such narrow radially excited states are still missing. More theoretical and experimental studies are needed.

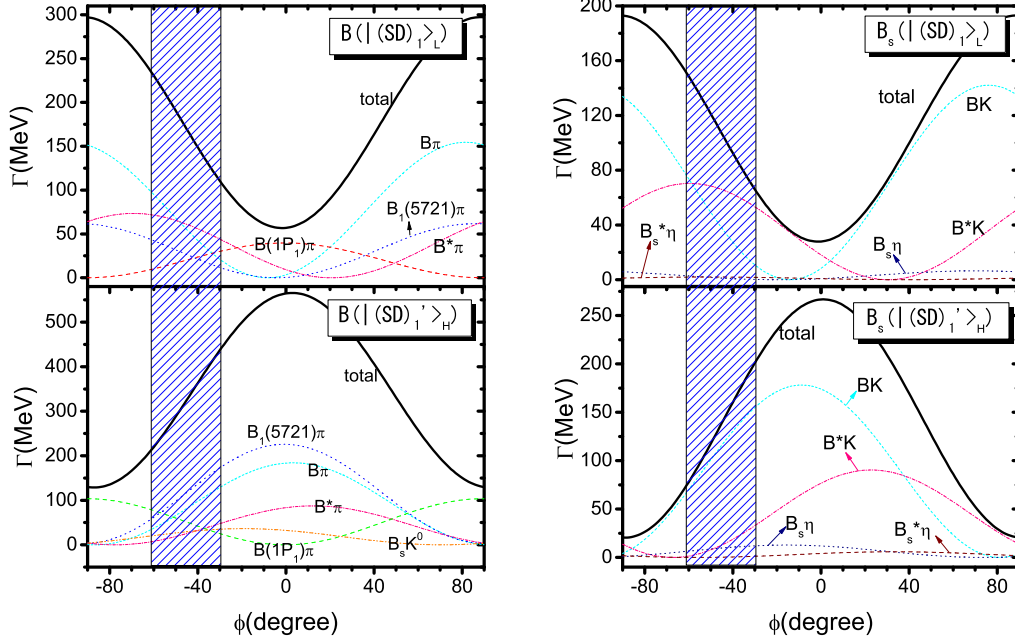


FIG. 1: The partial decay width and total decay width for the mixed states via 2^3S_1 - 1^3D_1 mixing in the B - and B_s -meson families as functions of the mixing angle ϕ . The shaded bands correspond to the possible mixing angle region derived from the strong decay properties of the $D(2600)$ and $D_{s1}(2700)$. The masses for the mixed states in the B -meson family, $B(|(SD)_1>_L>)$ and $B(|(SD)'_1>_H>)$, are adopted as 5920 and 6025 MeV, respectively, while, the masses for the mixed states in the B_s -meson family, $B_s(|(SD)_1>_L>)$ and $B_s(|(SD)'_1>_H>)$, are adopted as 6000 and 6100 MeV, respectively. The mass of $B(1P_1)$ is taken as 5720 MeV. In the figure, we have hidden some decay channels because of their small partial decay widths.

B. The 2^3S_1 - 1^3D_1 mixing

In 2010, the *BABAR* Collaboration discovered a new excited D -meson state $D(2600)$ in $D\pi$ and $D^*\pi$ decay channels with a mass of $M = 2612 \pm 6$ MeV and a total decay width of $\Gamma = 93 \pm 6 \pm 13$ MeV [1], which might be a partner of the $D_{s1}(2700)$ observed previously by *BABAR* and Belle [3, 4]. The $D(2600)$ may correspond to the $D(2650)$ observed by LHCb very recently [2].

In our previous work [8, 9], we have carefully studied the strong decay properties of the $D(2600)$ and $D_{s1}(2700)$. According to our analysis, both $D(2600)$ and $D_{s1}(2700)$ could be explained as the mixed state $|(SD)_1>_L$ via the 2^3S_1 - 1^3D_1 mixing:

$$\begin{pmatrix} |(SD)_1>_L \\ |(SD)'_1>_H \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} |2^3S_1> \\ |1^3D_1> \end{pmatrix}. \quad (9)$$

The mixing angle for the $D(2600)$ is $\phi \simeq -(36 \pm 6)^\circ$, while that for the $D_{s1}(2700)$ is $\phi = (-54 \pm 7)^\circ$. To explain the strong decay properties of the $D(2600)$ and/or $D_{s1}(2700)$, configuration mixing between 2^3S_1 and 1^3D_1 is also suggested in the literature [17, 26, 28].

Following the mixing scheme in Eq. (9), the strong de-

cay properties of the physical partners of the $D(2600)$ and $D_{s1}(2700)$, i.e., $|(SD)'_1>_H$, have been obtained as well. Taking the mass of $D(|(SD)'_1>_H)$ in the range of $(2.65 \sim 2.80)$ GeV, we predict that the $D(|(SD)'_1>_H)$ is a broad state with a width of $\Gamma \simeq (360 \pm 120)$ MeV. Its strong decays are dominated by $D\pi$ and $D_1(2420)\pi$. On the other hand, estimating the mass of $D_s(|(SD)'_1>_H)$ in the range of $(2.72 \sim 2.88)$ GeV, the total width is $\Gamma \simeq (120 \pm 10)$ MeV [8]. The predicted width for the $D_s(|(SD)'_1>_H)$ is not very broad. This state might be observed in the DK channel. It is interestingly found that the LHCb Collaboration has observed a new resonance $D_{s1}(2860)$ with a width of $\Gamma = (159 \pm 122)$ MeV very recently [52, 53]. Their analysis shows that the spin-parity should be $J^P = 1^-$. This newly observed resonance $D_{s1}(2860)$ can be favorably assigned as the physical partners of the $D_{s1}(2700)$, i.e., $D_s(|(SD)'_1>_H)$, predicted in our previous work [8].

If both $D(2600)$ and $D_{s1}(2700)$ indeed correspond to the mixed state $|(SD)_1>_L$, the 2^3S_1 - 1^3D_1 mixing might be a common character in the heavy-light mesons. The future search for these missing mixing states $|(SD)_1>_L$ and $|(SD)'_1>_H$ in the heavy-light meson spectroscopy becomes interesting and important.

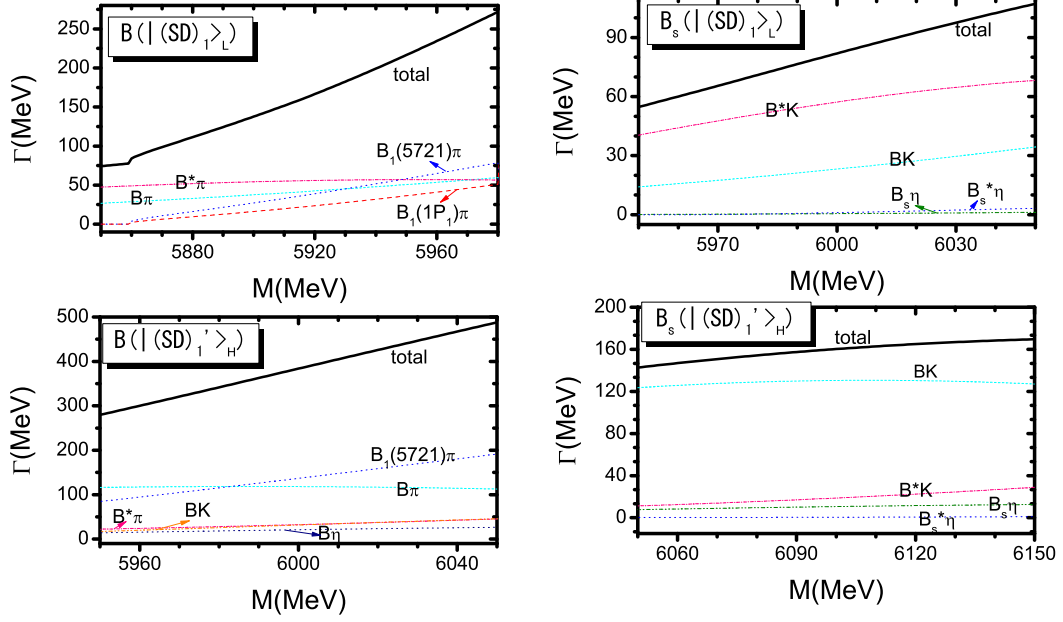


FIG. 2: The partial decay width and total decay width for the mixed states via 2^3S_1 - 1^3D_1 mixing in the B - and B_s -meson families as functions of the mass. The mixing angle is fixed with $\phi = -40^\circ$. In the figure, we have hidden some decay channels because of their small partial decay widths.

Firstly, we consider the 2^3S_1 - 1^3D_1 mixing in the B -meson family. According to the theoretical calculations of the B -meson spectroscopy, the mass of the low-mass state $|(SD)_1\rangle_L$ is $M \sim 5.9$ GeV [39, 40]. Thus, we set the mass of the low-mass mixed state $|(SD)_1\rangle_L$ with $M = 5.9$ GeV, and plot its partial decay widths and total decay width as functions of the mixing angle ϕ in Fig. 1. Theoretically, the mixing angle ϕ between 2^3S_1 and 1^3D_1 in the D -, D_s -, B - and B_s -meson families should be the same in the heavy quark symmetry limit. Combining our previous analysis of the strong decay properties of the $D(2600)$ and $D_{s1}(2700)$ [8, 9], the mixing angle might be $\phi \simeq -(45 \pm 16)^\circ$. From Fig. 1, it is seen that the mixed state $B(|(SD)_1\rangle_L)$ has a width of $\Gamma \simeq (150 \pm 50)$ MeV. Its decays are dominated by the $B\pi$ and $B^*\pi$; the partial width ratio between $B\pi$ and $B^*\pi$ is

$$\frac{\Gamma[B\pi]}{\Gamma[B^*\pi]} \simeq 1.0 \pm 0.3. \quad (10)$$

For the high-mass mixed state $B(|(SD)_1\rangle_H)$, we take its mass with $M = 6.0$ GeV, and plot its partial decay widths and total decay width as functions of the mixing angle ϕ in Fig. 1 as well, where we can see that this resonance is a broad state with a width of $\Gamma = (310 \pm 90)$ MeV. It decays mainly through the $B\pi$ and $B_1(5721)\pi$ channels.

Finally, we study the 2^3S_1 - 1^3D_1 mixing in the B_s -meson family. According to the theoretical predictions in various models, the masses for the mixed states $|(SD)_1\rangle_L$ and $|(SD)_1\rangle_H$ are around 6.0 and 6.1 GeV, respectively [39, 40]. With these predicted masses, we plot the partial decay widths and to-

tal decay widths for the mixed states $|(SD)_1\rangle_L$ and $|(SD)_1\rangle_H$ as functions of the mixing angle ϕ in Fig. 1, respectively. Adopting the mixing angle $\phi \simeq -(45 \pm 16)^\circ$, from Fig. 1 we see that the low-mass state $|(SD)_1\rangle_L$ might have a width of $\Gamma = (130 \pm 50)$ MeV. Its strong decays are dominated by the BK and B^*K channels. The high-mass state $|(SD)_1\rangle_H$ is a slightly broader state with a width of $\Gamma = (140 \pm 60)$ MeV. The decay widths for the $|(SD)_1\rangle_L$ and $|(SD)_1\rangle_H$ are sensitive to the mixing angle.

The predicted masses of these mixed states $|(SD)_1\rangle_L$ and $|(SD)_1\rangle_H$ in B - and B_s -meson spectroscopy have a large uncertainty, which may bring uncertainties to the predicted decay widths. To investigate this effect, we plot the decay width as a function of the mass in Fig. 2 with the mixing angle $\phi = -40^\circ$. It shows that there is an uncertainty of about $(50 \sim 200)$ MeV in the total decay width for the uncertainties of the mass. The sensitivity of different decay modes to the mass can also be seen clearly in the plot.

In summary, configuration mixing might exist between 2^3S_1 and 1^3D_1 according to our model prediction. The decay properties are sensitive to the mixing angle. As flavor partners of the $D(2600)$ and $D_{s1}(2700)$, the low-mass mixed states $B(|(SD)_1\rangle_L)$ and $B_s(|(SD)_1\rangle_L)$ have a relatively narrow width and are most likely to be observed in future experiments. In the high-mass mixed states, the widths of charmed-strange state $D_s(|(SD)_1\rangle_H)$ and bottom-strange state $B_s(|(SD)_1\rangle_H)$ are as comparable as those of the low-mass mixed states, which are possibly to be found in experiments as well, while the bottom state $B(|(SD)_1\rangle_H)$ is a broad state, which might be hard

to find in experiments. However, it should be pointed out that in some Refs. [10, 11, 18–20], one believes that configuration mixing might be weak between 2^3S_1 and 1^3D_1 . Within their models, the strong decay properties of the $D(2600)$ and/or $D_{s1}(2700)$ can be well explained by taking them as a pure 2^3S_1 state. To uncover the puzzles in the 2^3S_1 and 1^3D_1 states, more accurate measurements for the $D(2600)$ and $D_{s1}(2700)$ as well as a further search for the missing resonances with $J^P = 1^-$ in the heavy-light meson spectroscopy in experiment is needed.

C. The 1^1D_2 - 1^3D_2 mixing

The $D(2750)$ is a new excitation observed by *BABAR* Collaboration in the $D\pi$ channel with a narrow width of $\Gamma \simeq 71 \pm 17$ MeV [1]. This state might correspond to the $D(2740)$ observed by the LHCb Collaboration in the same channel recently [2]. Their helicity analysis indicates that this state should have an unnatural parity, i.e., $J^P = 0^-, 1^+, 2^-, \dots$. In our previous work [9], the new resonance $D(2750)$ was discussed carefully. We concluded that the spin and parity numbers of the $D(2750)$ might be $J^P = 2^-$, this state is most likely to be the high-mass mixed state $|1D_2'\rangle_H$ via the 1^1D_2 - 1^3D_2 mixing:

$$\begin{pmatrix} |1D_2\rangle_L \\ |1D_2'\rangle_H \end{pmatrix} = \begin{pmatrix} \cos \phi_{1D} & \sin \phi_{1D} \\ -\sin \phi_{1D} & \cos \phi_{1D} \end{pmatrix} \begin{pmatrix} |1^1D_2\rangle \\ |1^3D_2\rangle \end{pmatrix}, \quad (11)$$

with a mixing angle $\phi_{1D} = -(51 \pm 18)^\circ$. The mixing angle is consistent with that ($\phi_{1D} = -50.8^\circ$) obtained in the heavy quark symmetry limit [27]. Our predictions are in compatible with the recent analysis in Ref. [10]. It should be mentioned that with the mixing angle $\phi_{1D} = -50.8^\circ$, the low-mass state $|1D_2\rangle_L$ and high-mass state $|1D_2'\rangle_H$ will correspond to a broad state and a narrow state, respectively [27, 29].

The strong decay properties of the low-mass state $D(|1D_2\rangle_L)$ have been studied in [9] as well. The mass of

$D(|1D_2\rangle_L)$ is likely to be ~ 2.7 GeV. This state is a broad state with a width of $\Gamma \simeq (250 \sim 500)$ MeV, and its strong decays are dominated by the $D^*\pi$ and $D_2(2460)\pi$ channels. The $D(|1D_2\rangle_L)$ might be hard to observe in experiments because of its broad width.

In our previous study [8], we also found that the 1^1D_2 - 1^3D_2 mixing might be crucial to uncovering the longstanding puzzles in the $D_{sJ}(2860)$. Many people believe that the $D_{sJ}(2860)$ might be the 1^3D_3 state. However, considering the $D_{sJ}(2860)$ as the 1^3D_3 state only, one can not understand the important partial width ratio of $\Gamma(DK)/\Gamma(D^*K)$ measured by *BABAR* [5] at all. Considering there are two D_s states with masses around 2.86 GeV, one resonance corresponds to the 1^3D_3 [denoted by $D_{sJ_1}(2860)$] and the other resonance is the mixed state $|1D_2'\rangle_H$ [denoted by $D_{sJ_2}(2860)$] via the 1^1D_2 - 1^3D_2 mixing with the same mixing angle, we can explain the present strong decay properties of the $D_{sJ}(2860)$ observed in experiments naturally [8]. Our conclusion is compatible with the recent theoretical analysis in Ref. [11].

If the $D_{sJ_2}(2860)$ could be assigned as the high-mass state $|1D_2'\rangle_H$ indeed, its low-mass partner $|1D_2\rangle_L$ might be observed in experiments as well. The mass of low-mass partner $|1D_2\rangle_L$ is estimated to be $30 \sim 50$ MeV lighter than that of the high-mass state [39, 40, 44]. Thus, the mass of $|1D_2\rangle_L$ might be in the range of $(2.80 \sim 2.83)$ GeV. Its strong decay properties have been analyzed in our previous work [9]. It is found $|1D_2\rangle_L$ is a broad state with a width of $\Gamma \simeq (260 \pm 30)$ MeV. The strong decays are dominated by the D^*K channel.

It should be emphasized that if both $D(2750)$ and $D_{sJ_2}(2860)$ indeed correspond to the mixed state $|1D_2'\rangle_H$, the 1^1D_2 - 1^3D_2 mixing might exist in the B - and B_s -meson spectroscopy as well. To provide useful clues for the future experimental search for these mixed states $|1D_2'\rangle_H$ and $|1D_2\rangle_L$ in B - and B_s -meson spectroscopy, we study their strong decay properties in the following.

Firstly, we study the 1^1D_2 - 1^3D_2 mixing in the B -meson family. According to the predictions in Refs. [39, 40, 44], the mass of the high-mass mixed state $|1D_2'\rangle_H$ is in the range of $(6.04 \sim 6.12)$ GeV. However, we find the masses of the $|1D_2'\rangle_H$ resonances might be systemically overestimated by ~ 100 MeV in theory, if the $D(2750)$ and $D_{sJ_2}(2860)$ correspond to the mixed state $|1D_2'\rangle_H$. Thus, the mass of the $|1D_2'\rangle_H$ in the B -meson family might be $M \simeq (5.94 \sim 6.02)$ GeV.

Taking the mass of $B(|1D_2'\rangle_H)$ with $M = 5.98$ GeV, we plot

the partial decay widths and total decay width as functions of the mixing angle ϕ_{1D} in Fig. 3. It is shown that the decay width of the high-mass state $|1D_2'\rangle_H$ is about $\Gamma \simeq (40 \sim 90)$ MeV, when we take the mixing angle with $\phi_{1D} = (-51 \pm 18)^\circ$. The main decay channels are $B^*\pi$ and $B_2(5747)\pi$. The predicted branching ratio is

$$\frac{\Gamma[B^*\pi]}{\Gamma_{\text{total}}} \simeq 70\% \sim 93\%. \quad (12)$$

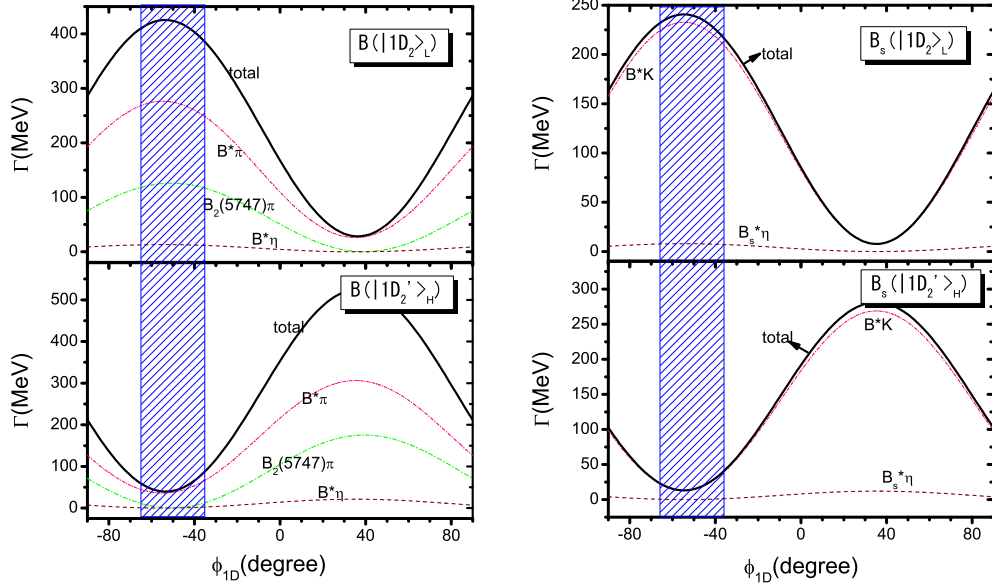


FIG. 3: The partial decay width and total decay width for the mixed states via 1^1D_2 - 1^3D_2 mixing in the B - and B_s -meson families as functions of the mixing angle ϕ_{1D} . The shaded region corresponds to possible mixing angle region derived from the strong decay properties of the $D(2750)$. The masses of the low-mass mixed states $|1D_2\rangle_L$ in the B - and B_s -meson families are adopted as 5.95 GeV and 6.05 GeV, respectively. The masses of the high-mass mixed states $|1D_2'\rangle_H$ in the B - and B_s -meson families are adopted as 5.98 GeV and 6.08 GeV, respectively. In the figure, we have hidden some decay channels because of their small partial decay widths.

The partial widths for the $B(1^3P_0)\pi$, $B(1P_1)\pi$, $B(1P_1')\pi$, $B^*\eta$ and B_s^*K channels are tiny and are not shown in the figure. Such a narrow state, $B(|1D_2'\rangle_H)$, is most likely to be observed in the $B^*\pi$ channel.

For the low-mass mixed state $|1D_2\rangle_L$, its mass might be (20 ~ 50) MeV lighter than that of the $|1D_2\rangle_H$ [39, 40, 44]. Thus, the mass of $|1D_2\rangle_L$ might be in the range $M = (5.9 \sim 6.0)$ GeV. The dependence of the strong decay width of the $|1D_2\rangle_L$ on the mass is shown in Fig. 4 as well. From the figure, we can see that the low-mass state $B(|1D_2\rangle_L)$ is a very broad state with a width of $\Gamma \simeq (400 \pm 20)$ MeV. Its strong decays are governed by the $B^*\pi$ and $B_2(5747)\pi$ channels. This broad state might be hard to observe in experiments.

Secondly, we study the 1^1D_2 - 1^3D_2 mixing in the B_s -meson family. According to the theoretical predictions, the mass gap between the B_s resonances and the B resonances is about 100 MeV [39, 40, 44]. Thus, the high-mass state $B_s(|1D_2'\rangle_H)$ might have a mass of $M \simeq (6.04 \sim 6.12)$ GeV, while the mass of the low-mass state $B_s(|1D_2\rangle_L)$ might be $M \simeq (5.99 \sim 6.10)$ GeV.

Taking the mass of the $B_s(|1D_2'\rangle_H)$ with $M = 6.08$ GeV, we plot the partial widths and total decay width as functions of the mixing angle in Fig. 3. Adopting the physical mixing angle $\phi_{1D} = (-51 \pm 18)^\circ$ determined before, we find the $B_s(|1D_2'\rangle_H)$ is a fairly narrow state with a width of $\Gamma \simeq (13 \sim 50)$ MeV. Its decays are dominated by the B^*K channel. The predicted

branching ratio is

$$\frac{\Gamma[B^*K]}{\Gamma_{\text{total}}} \simeq 100\%. \quad (13)$$

This narrow state is most likely to be discovered in the B^*K channel in future experiments.

Furthermore, we analyze the strong decay properties of the low-mass state $B_s(|1D_2\rangle_L)$. Its partial decay widths and total width as functions of the mixing angle are plotted in Fig. 3 as well, where we have fixed the mass with $M = 6.05$ GeV. From the figure, we find that the low-mass state $B_s(|1D_2\rangle_L)$ has a broad width of $\Gamma \simeq 240$ MeV, and its decays are governed by the B^*K channel.

Finally, we study the effects of the mass uncertainties of the mixing states $|1D_2\rangle_L$ and $|1D_2'\rangle_H$ in B and B_s spectroscopy on the decay widths. To investigate this effect, we plot the decay width as a function of the mass in Fig. 4 with the mixing angle fixed at $\phi_{1D} = -50.8^\circ$. The sensitivity of different decay modes to the mass can also be seen clearly in the plot.

As a whole, the high-mass mixed states $B(|1D_2'\rangle_H)$ and $B_s(|1D_2'\rangle_H)$ might be narrow states, which might be observed in the $B^*\pi$ and B^*K channels, respectively. However, the low-mass mixed state $|1D_2\rangle_L$ should be a broad state. The typical total decay widths for the low-mass mixed states $D(|1D_2\rangle_L)$ and $B(|1D_2\rangle_L)$ are usually larger than 300 MeV, which might be too broad to be observed in experiments. However, those low-mass mixed states containing a strange quark, $D_s(|1D_2\rangle_L)$

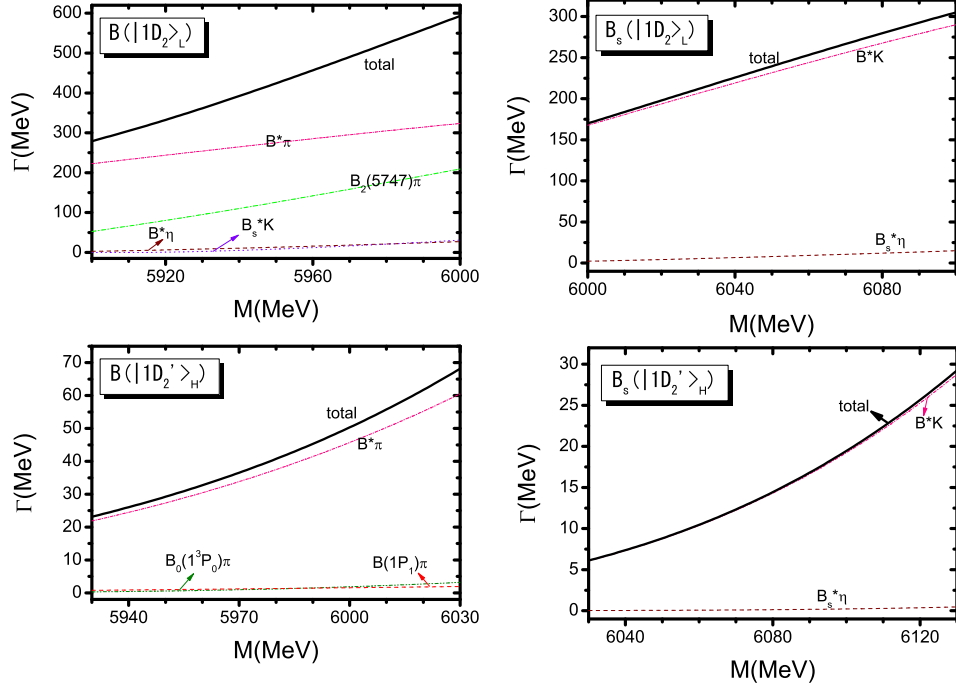


FIG. 4: The partial decay width and total decay width for the mixed states via 1^1D_2 - 1^3D_2 mixing in the B - and B_s -meson families as functions of the mass. The mixing angle is adopted as $\phi_{1D} = -50.8^\circ$. In the figure, we have hidden some decay channels because of their small partial decay widths.

and $B_s(|1D_2\rangle_L)$, have a relatively narrower width, ~ 250 MeV, which have some possibilities to be observed in future experiments.

D. The 1^3D_3 states

The $D(2760)$ is a new excitation in the D -meson family observed by *BABAR* in the $D\pi$ channel with a mass of $M = 2763.3 \pm 4.6$ MeV and a width of $\Gamma = 60.9 \pm 8.7$ MeV. Although its mass and width are very close to those of the $D(2750)$, they might be two different resonances due to several reasons, which have been explained in Ref. [9]. Recently, this state was confirmed by the LHCb Collaboration in the $D\pi$ and $D^*\pi$ channels, and their helicity analysis indicates that this state should have a natural parity, i.e., $J^P = 0^+, 1^-, 2^+, 3^-, \dots$. In our previous work [9], this new resonance $D(2760)$ was studied carefully. It is predicted that the $D(2760)$ could be assigned to the 1^3D_3 ($J^P = 3^-$) state. Our conclusion is consistent with that in Refs. [10, 18, 23]. Taking the $D(2760)$ as the assignment of the 1^3D_3 , the total decay width is $\Gamma \simeq 68$ MeV, which is consistent with the data. Its strong decays are dominated by the $D\pi$ and $D^*\pi$ channels, and the partial width ratio is

$$\frac{\Gamma[D^*\pi]}{\Gamma[D\pi]} \simeq 0.65, \quad (14)$$

which can be tested in future experiments.

Furthermore, in our previous work [8], we suggested there are two largely overlapping resonances at 2.86 GeV in the D_s -meson family. One state corresponds to the 1^3D_3 ($J^P = 3^-$) state [denoted by $D_{sJ_1}(2860)$], which dominantly decays into the DK channel, while the other one corresponds to the mixed state $|1D_2'\rangle_H$ with $J^P = 2^-$ [denoted by $D_{sJ_2}(2860)$], which dominantly decays into the D^*K channel. Considering the $D_{sJ_1}(2860)$ as the 1^3D_3 state, we have studied its strong decay properties. The predicted decay width is $\Gamma \simeq 45$ MeV, which is highly comparable with the experimental data ($\Gamma = 58 \pm 11$ MeV). This state has two main decay channels DK and D^*K , and the predicted partial width ratio is

$$\frac{\Gamma[D^*K]}{\Gamma[DK]} \simeq 0.5, \quad (15)$$

which can be tested in future experiments. The present experimental measurement of the ratio, $\Gamma[D^*K]/\Gamma[DK] = 1.10 \pm 0.15 \pm 0.19$, might include the contributions of both $D_{sJ_1}(2860)$ and $D_{sJ_2}(2860)$. Thus, its value is obviously larger than the theoretical predictions by assuming the $D_{sJ_1}(2860)$ as the 1^3D_3 state only. If both $D(2760)$ and $D_{sJ_1}(2860)$ correspond to the 1^3D_3 excitation indeed, their flavor partners in B and B_s spectroscopy should be observed in future experiments as well.

In the B -meson family, the predicted mass of 1^3D_3 is around 6.0 \sim 6.1 GeV [33, 39, 40, 44]. Considering the $D(2760)$ and $D_{sJ_1}(2860)$ as the 1^3D_3 assignment, their mass is systemically overestimated by 50 \sim 100 MeV in theory. Thus, we estimate the mass of $B(1^3D_3)$ is in the range of

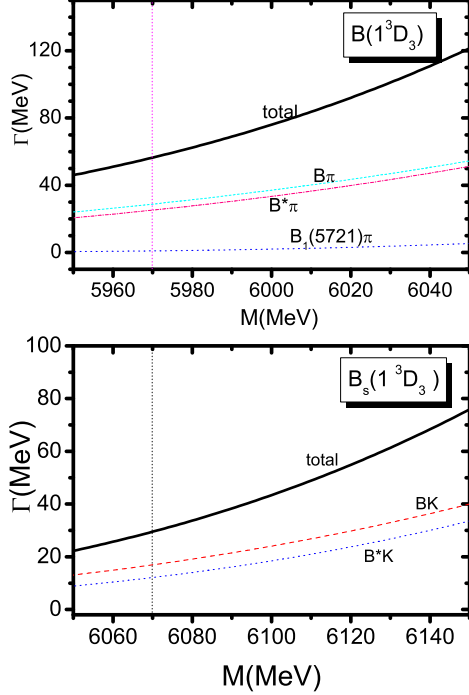


FIG. 5: The partial decay widths and total decay width of the 1^3D_3 excitations in the B - and B_s -meson families as functions of the mass. In the figure, we have hidden some decay channels because of their small partial decay widths.

(5.95 ~ 6.05) GeV. We calculate the strong decay properties of $B(1^3D_3)$, which are shown in Fig. 5. From the figure, it is seen that the $B(1^3D_3)$ is a narrow state with a width of $\Gamma \simeq 50 \sim 120$ MeV. Its decays are dominated by the $B\pi$ and $B^*\pi$ channels. The partial width ratio between $B\pi$ and $B^*\pi$ is insensitive to the mass, and its predicted value is

$$\frac{\Gamma[B^*\pi]}{\Gamma[B\pi]} \simeq 0.9. \quad (16)$$

Recently, the CDF Collaboration claimed that they found

some evidence of a new resonance $B(5970)$ in the $B\pi$ channel [6]. The central value of the width is $\Gamma \sim 70$ MeV. From Fig. 5, it is interestingly found that the $B(5970)$ is most likely to be the 1^3D_3 assignment. Considering the $B(5970)$ as the 1^3D_3 state, the predicted decay width is $\Gamma \simeq 60$ MeV, and the $B\pi$ and $B^*\pi$ decay channels are its dominant decay modes. Our predictions are in a good agreement with the observations. Although the mass of the $B(5970)$ is close to the estimated masses of the mixed states $B(|(SD)_1\rangle_L)$ and $B(|1D'_2\rangle_H)$, they are not good candidates for the $B(5970)$. For the $B(|(SD)_1\rangle_L)$, when we set its mass with ~ 5.97 GeV, its theoretical width is ~ 300 MeV, which is too large to be compared with the width of the $B(5970)$, while for the $B(|1D'_2\rangle_H)$, its strong decays are governed by the $B^*\pi$, and the $B\pi$ channel is forbidden. Thus, it could not be assigned to the $B(5970)$.

Finally, we study the strong decays of 1^3D_3 in the B_s -meson family. Usually, the mass of the B_s resonances is about 100 MeV larger than that of the B resonances [39, 40, 44]. Thus, we estimate the mass of $B_s(1^3D_3)$ might be in the range of (6.05 ~ 6.15) GeV. We have studied the strong decay properties of $B_s(1^3D_3)$, which are shown in Fig. 5. It is found that the $B_s(1^3D_3)$ might be a narrow state with a width of $\Gamma \simeq (25 \sim 75)$ MeV, and its strong decays are dominated by the BK and B^*K channels. If the $B(5970)$ corresponds to the $B(1^3D_3)$ indeed, the mass of the $B_s(1^3D_3)$ might be $M \simeq 6.07$ GeV. With this mass, the predicted width for the $B_s(1^3D_3)$ is

$$\Gamma \simeq 30 \text{ MeV}, \quad (17)$$

and the predicted ratio between BK and B^*K is

$$\frac{\Gamma[BK]}{\Gamma[B^*K]} \simeq 1.4. \quad (18)$$

We hope the experimenters can carry out a search for the $B_s(1^3D_3)$ in the BK and B^*K channels, which is also helpful to clarify the nature of the newly observed state $B(5970)$.

In brief, the $D(2760)$, $D_{sJ_1}(2860)$ and $B(5970)$ might be classified as the low-lying D -wave excitations with $J^P = 3^-$ (i.e., 1^3D_3). The last unobserved one in the B_s -meson family should be a narrow state, which is most likely to be found in the BK and B^*K channels.

E. The 2^3P_0 states

In the D -meson family, the predicted mass for the 2^3P_0 state is ~ 2.95 GeV [34, 39, 40]. For the uncertainties of the predicted mass, we vary the mass of $D(2^3P_0)$ in the range of $M = (2.9 \sim 3.0)$ GeV, and plot the strong decay properties in Fig. 6. From the figure it is seen that the $D(2^3P_0)$ might be a very broad state with a width of $\Gamma \simeq 600 \sim 800$ MeV. This state dominantly decays into the $D(2550)\pi$, $D\pi$ and $D(2430)\pi$ channels. A very broad width for the $D(2^3P_0)$ was also pre-

dicted in a recent study with the 3^3P_0 model [10]. Such a broad resonance might be difficult to observe in experiments.

Recently, the LHCb Collaboration observed a new excited D -meson state $D_J^*(3000)$ in the $D\pi$ channel with a rather narrow width, $\Gamma \simeq 110$ MeV [2]. Their further helicity analysis indicates that this new state should have a natural parity, that is, the spin-parity number of the $D_J^*(3000)$ should be $J^P = 0^+, 1^-, 2^+, \dots$. In Ref. [34], Sun *et al.* believed that the $D_J^*(3000)$ could be explained as the $D(2^3P_0)$. If we exclude the contribution of the $D(2550)\pi$ channel to the decay width,

the 2^1P_1 - 2^3P_1 mixing:

$$\begin{pmatrix} |2P_1\rangle_L \\ |2P_1'\rangle_H \end{pmatrix} = \begin{pmatrix} \cos\phi_{2P} & \sin\phi_{2P} \\ -\sin\phi_{2P} & \cos\phi_{2P} \end{pmatrix} \begin{pmatrix} |2^1P_1\rangle \\ |2^3P_1\rangle \end{pmatrix}, \quad (19)$$

with a mixing angle $\phi_{2P} \simeq -(51 \pm 27)^\circ$. This mixing angle is consistent with that ($\phi_{2P} = -54.7^\circ$) derived in the heavy quark symmetry limit [27]. It should be mentioned that with the mixing angle $\phi_{2P} = -54.7^\circ$, the low-mass state $|2P_1\rangle_L$ usually has a broad width, while the high-mass state $|2P_1'\rangle_H$ has a narrower width [27, 29]. As a broad mixed state, the strong decays of the $D_{sJ}(3040)$ are dominated by the D^*K channel. The partial widths of $D_0(2400)K$, $D_1(2430)K$, and $D_2(2460)K$ are sizable as well. Our conclusion is consistent with that obtained in Refs. [16, 21, 40].

As the physical partner of $|2P_1\rangle_L$, the high-mass state $|2P_1'\rangle_H$ might be observed in experiments. Assuming the mass of the $|2P_1'\rangle_H$ is in the range of $(3.04 \sim 3.2)$ GeV, its strong decay properties have been analyzed in our previous work [8]. The decay width of $|2P_1'\rangle_H$ is narrower than that of the $D_{sJ}(3040)$, which increases fast with the increasing mass. The main decay channels might be $D_0(2400)K$, $D_1(2430)K$ and DK^* .

If the $D_{sJ}(3040)$ could indeed be assigned as a mixed state via the 2^1P_1 - 2^3P_1 mixing, the other mixed states between 2^1P_1 and 2^3P_1 in the D -, B - and B_s -meson spectroscopy might be found in future experiments.

Firstly, we study the mixed states of 2^1P_1 - 2^3P_1 in the D -meson family. According to the predictions in Refs. [39, 40, 44], the mass of the low-mass state $D(|2P_1\rangle_L)$ might be in the range of $(2.9 \sim 3.0)$ GeV, while the mass of the high-mass state $D(|2P_1'\rangle_H)$ might be in the range of $(3.0 \sim 3.1)$ GeV. Adopting the mixing angle $\phi_{2P} = -54.7^\circ$ predicted in the heavy quark symmetry limit, we have plotted their partial decay widths and total decay width as functions of the mass in Fig. 7. From the figure, it is seen that the low-mass state $D(|2P_1\rangle_L)$ is a broad state with a width of $\Gamma \simeq (370 \pm 90)$ MeV, its strong decays are dominated by the $D^*\pi$ and $D(2600)\pi$ channels, and the partial decay widths of the $D_0(2400)\pi$, $D_1(2430)\pi$ and D_s^*K channels are sizable as well.

Although the high-mass state $D(|2P_1'\rangle_H)$ has a relatively narrower width than the low-mass state $D(|2P_1\rangle_L)$, the $D(|2P_1'\rangle_H)$ is also a very broad state with a width of $\Gamma \simeq (300 \pm 90)$ MeV when we adopt its mass in the range of $(3.0 \sim 3.1)$ GeV. Its strong decays are governed by the $D(2600)\pi$ channel. The other channels $D\rho$, $D_0(2400)\pi$, $D_{s0}(2317)K$ and $D_1(2430)\pi$ also have sizable contributions to the decays.

Recently, a new resonance $D_J(3000)$ was observed by the LHCb. Its measured mass and width are $M \simeq 2972$ MeV and $\Gamma \simeq 188 \pm 45$ MeV, respectively. Furthermore, the helicity analysis from LHCb indicates that this state has an unnatural parity, that is, its spin-parity should be $J^P = 0^-, 1^+, 2^-, \dots$. According to the observed mass, the observed $D^*\pi$ decay mode and possible J^P numbers of the $D_J(3000)$, we predict that the $D_J(3000)$ might be a good candidate for the low-mass state $D(|2P_1\rangle_L)$. As the $D(|2P_1\rangle_L)$ candidate with the mixing angle $\phi_{2P} = -54.7^\circ$ derived in the heavy quark symmetry limit, the $D_J(3000)$ might have a broad width of $\Gamma \simeq 360$ MeV (see Fig. 7), which is about a factor 2 larger than the data

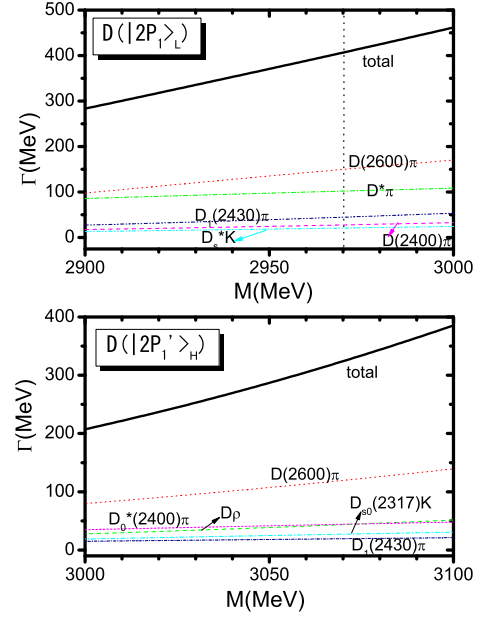


FIG. 7: The partial decay widths and total decay width of the mixed states via the 2^1P_1 - 2^3P_1 mixing in the D -meson family as functions of the mass. Here, the mixing angle $\phi_{2P} = -54.7^\circ$ is adopted. In the figure, we have hidden some decay channels because of their small partial decay widths.

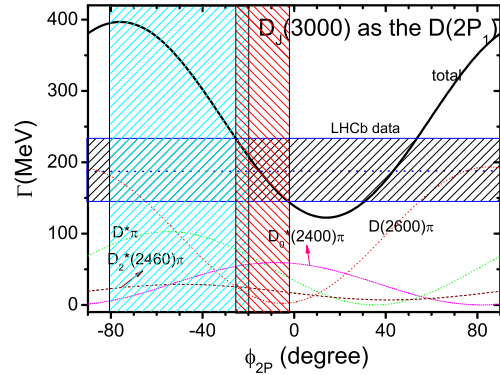


FIG. 8: The variation the decay properties of the mixed state $|2P_1\rangle_L$ as a candidate of $D_J(3000)$ with the mixing angle ϕ_{2P} . The left shaded band corresponds to the possible mixing angle region derived from the strong decay properties of the $D_{sJ}(3040)$, while the right shaded band corresponds to the mixing angle region derived from the strong decay properties of the $D_J(3000)$. In the figure, we have hidden some decay channels for their small partial decay widths.

($\Gamma \simeq 188 \pm 45$ MeV). The predicted partial decay width ratio of the main two decay channels $D^*\pi$ and $D(2600)\pi$ is

$$\frac{\Gamma[D^*\pi]}{\Gamma[D(2600)\pi]} \simeq 0.7. \quad (20)$$

However, we have noticed that the mixing angle ϕ_{2P} deter-

mined by the strong decay properties of the $D_{sJ}(3040)$ bares a large uncertainty. Taking the $D_J(3000)$ as a candidate of the $D(|2P_1\rangle_L)$ state, we have plotted the partial widths and total decay width as functions of the mixing angle ϕ_{2P} in Fig. 8. From the figure, it is found that the decay width is very sensitive to the mixing angle. If we adopt a mixing angle in the range of $\phi_{2P} = -(3 \sim 26)^\circ$, the decay width of the $D_J(3000)$ can be explained. The mixing angle for $D_J(3000)$ has an overlap with that for the $D_{sJ}(3040)$ in the range of $\phi_{2P} = -(20 \sim 26)^\circ$. It is found that the mixing angle $\phi_{2P} = -(20 \sim 26)^\circ$ obtained in present work is close to the mixing angle $\phi_{2P} = -30.4^\circ$ suggested in [11].

If both $D_{sJ}(3040)$ and $D_J(3000)$ can be assigned as the mixed state $|2P_1\rangle_L$, they should share nearly the same mix-

ing angle. Taking the same mixing angle we predict that the width of the $D_J(3000)$ should be a little larger than that of the $D_{sJ}(3040)$ for the larger decay phase space and more decay channels of the $D_J(3000)$.

Recently, the strong decay properties of the $D_J(3000)$ were studied in Ref. [34] as well. Considering the $D_J(3000)$ as the $D(|2P_1\rangle_L)$ with the mixing angle $\phi_{2P} = -54.7^\circ$, they could well explain the measured width of the $D_J(3000)$. However, it should be pointed out that they did not include the $D(2600)\pi$ decay mode in their calculations. Excluding the $D(2600)\pi$, our prediction is in agreement with that in Ref. [34]. To understand the nature of the $D_J(3000)$, more observations are needed in future experiments.

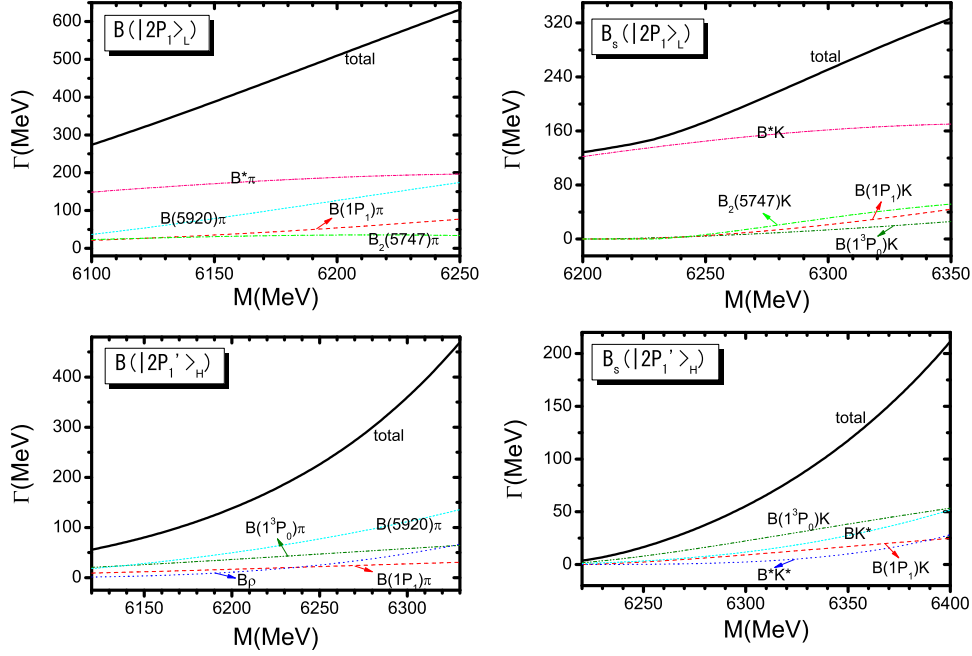


FIG. 9: The decay width of the mixed states via the 2^1P_1 - 2^3P_1 mixing in the B - and B_s -meson families as a function of the mass. Here, the mixing angle $\phi_{2P} = -54.7^\circ$ is adopted. The $B(5920)\pi$ stands for the mixed state $B(|(SD)_1\rangle_L)\pi$. In the figure, we have hidden some decay channels because of their small partial decay widths.

In the B -meson family, the predicted mass for the low-mass state $B(|2P_1\rangle_L)$ is $M \simeq 6.2$ GeV [39, 40, 44], the mass of the high-mass state $B(|2P'_1\rangle_H)$ is about $(20 \sim 80)$ MeV heavier than that of the low-mass state. Considering the uncertainty of the predicted masses for these states, in Fig. 9 we plot the partial decay widths and total decay width of $B(|2P_1\rangle_L)$ and $B(|2P'_1\rangle_H)$ as functions of the mass in the possible range, where we adopt the mixing angle $\phi_{2P} = -54.7^\circ$ derived in the heavy quark symmetry limit. From the figure, we find

that the low-mass state $B(|2P_1\rangle_L)$ is a broad state with a width of $\Gamma \simeq (450 \pm 150)$ MeV. The $B^*\pi$ decay mode governs its strong decays, while the other decay channels $B(|(SD)_1\rangle_L)\pi$, $B(1P_1)\pi$, $B(1^3P_2)\pi$, B_s^*K , $B(1^3P_0)\pi$ and $B^*\eta$ also have sizable partial decay widths.

The width of the high-mass state $B(|2P'_1\rangle_H)$, $\Gamma \simeq (50 \sim 450)$ MeV, is much narrower than that of the low-mass state $B(|2P_1\rangle_L)$. The $B(|2P'_1\rangle_H)$ dominantly decays into $B(|(SD)_1\rangle_L)\pi$, $B(1^3P_0)\pi$, $B(1P_1)\pi$ and $B(1^3P_2)\pi$. It might be

a challenge to observe the $B(|2P_1'\rangle_H)$ in these final states with higher excitations of the B meson.

Since the $D_{sJ}(3040)$ as a broad state has been observed in experiments, the broad resonance $B(|2P_1\rangle_L)$ might be observed in the $B^*\pi$ as well. To further know about the effects of the uncertainties of the mixing angle on the strong decays of $B(|2P_1\rangle_L)$, we have plotted the partial widths and total decay width of $B(|2P_1\rangle_L)$ as functions of the mixing angle ϕ_{2P} in Fig. 10, where we have fixed the mass of $B(|2P_1\rangle_L)$ with

$M = 6175$ MeV predicted in [39]. From the figure, it is seen that even the mixing angle ϕ_{2P} varies in a large range $\phi_{2P} \simeq -20^\circ \sim -80^\circ$, the strong decays of $B(|2P_1\rangle_L)$ are still dominated by the $B^*\pi$ channel, and the uncertainty of the decay width is no more than 60 MeV.

In the B_s -meson family, the predicted mass for the low-mass state $B_s(|2P_1\rangle_L)$ is $M \simeq (6.28 \sim 6.32)$ GeV [39, 40, 44], the mass of the high-mass state $B_s(|2P_1'\rangle_H)$ is about $(20 \sim 30)$ MeV heavier than that of the low-mass state.

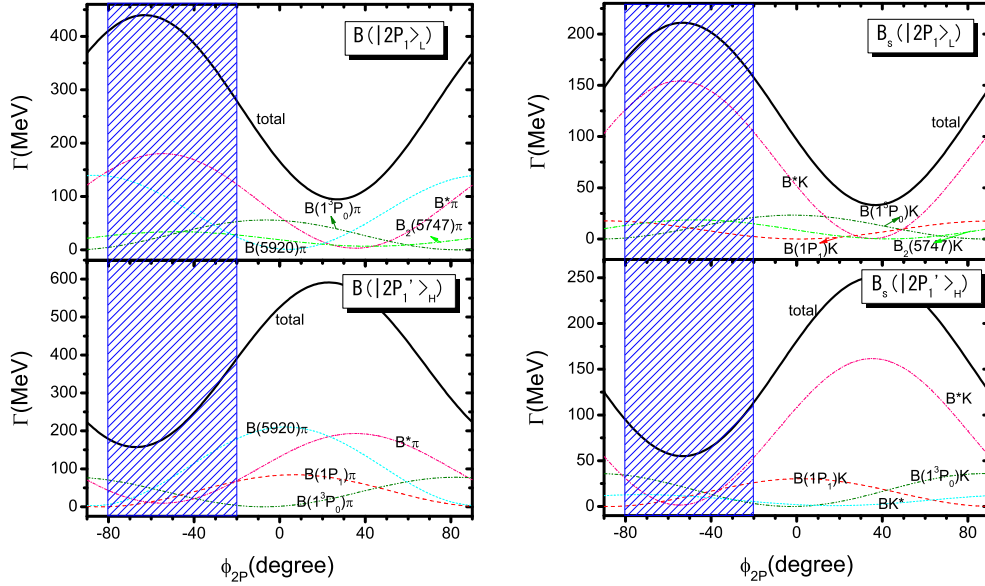


FIG. 10: The partial decay width and total decay width for the mixed states via 2^1P_1 - 2^3P_1 mixing in the B - and B_s -meson families as functions of the mixing angle ϕ_{2P} . The shaded region corresponds to the possible mixing angle region derived from the strong decay properties of the $D_{sJ}(3040)$ and $D_J(3000)$. The masses of the low-mass mixed states $|2P_1\rangle_L$ in the B - and B_s -meson families are adopted as 6.175 GeV and 6.275 GeV, respectively. The masses of the high-mass mixed states $|2P_1'\rangle_H$ in the B - and B_s -meson families are adopted as 6.225 GeV and 6.30 GeV, respectively. In the figure, we have hidden some decay channels because of their small partial decay widths.

We have plotted the partial decay widths and total decay width of $B_s(|2P_1\rangle_L)$ and $B_s(|2P_1'\rangle_H)$ as functions of the mass with the mixing angle $\phi_{2P} = -54.7^\circ$ in Fig. 9. From the figure, we find that the low-mass state $B_s(|2P_1\rangle_L)$ has a broad width of $\Gamma \simeq (220 \pm 70)$ MeV within the predicted mass region $M \simeq (6.28 \sim 6.32)$ GeV. The B^*K decay channel governs its strong decays, while the other decay channels $B(1P_1)K$, $B(1^3P_2)K$, $B(1^3P_0)K$ also have sizable partial decay widths. The width of the high-mass state $B_s(|2P_1'\rangle_H)$ is $\Gamma \simeq (95 \pm 33)$ MeV within the possible mass region $M \simeq (6.30 \sim 6.35)$ GeV, which is much narrower than that of $B_s(|2P_1\rangle_L)$. The $B_s(|2P_1'\rangle_H)$ dominantly decays into the $B(1^3P_0)K$, $B(1P_1)K$, and BK^* channels. Although the $B_s(|2P_1'\rangle_H)$ has a relatively

narrow width, it might be a great challenge to observe this resonance in these final states with higher B_s resonances.

The decay width of $B_s(|2P_1\rangle_L)$ is comparable with that of the $D_{sJ}(3040)$. Thus, the $B_s(|2P_1\rangle_L)$ is most likely to be observed in the B^*K channel. To further know about the effects of the uncertainties of the mixing angle on the strong decays of $B_s(|2P_1\rangle_L)$, we have plotted the partial widths and total decay width of $B_s(|2P_1\rangle_L)$ as functions of the mixing angle ϕ_{2P} in Fig. 10, where we have fixed the mass of $B_s(|2P_1\rangle_L)$ with $M = 6275$ MeV. From the figure, it is seen that even the mixing angle ϕ_{2P} is changed in a large range $\phi_{2P} \simeq -20^\circ \sim -80^\circ$, the strong decays of $B_s(|2P_1\rangle_L)$ are still dominated by the B^*K channel, and the uncertainty of the decay width is no more

than 50 MeV.

In summary, the strong decay properties of both $D_{sJ}(3040)$ and $D_J(3000)$ can be explained by assigning them as the low-mass mixed state $|2P_1\rangle_L$ via the 2^1P_1 - 2^3P_1 with a mixing angle in the range of $\phi_{2P} = -(20 \sim 26)^\circ$. The mixed state $B_s(|2P_1\rangle_L)$ is a narrow state, which is most likely to be observed in the B^*K channel. The mixed state $B(|2P_1\rangle_L)$ might

have a broad width; thus, its discovery potentials might be small. The high-mass mixed states $D(|2P_1'\rangle_H)$, $D_s(|2P_1'\rangle_H)$, $B(|2P_1'\rangle_H)$, and $B_s(|2P_1'\rangle_H)$ are usually narrower than those of low-mass states. These states dominantly decay into the first P -wave states by emitting a light pseudoscalar meson. Observations in the final states containing a low-lying P -wave state are expected to be carried out in future experiments.

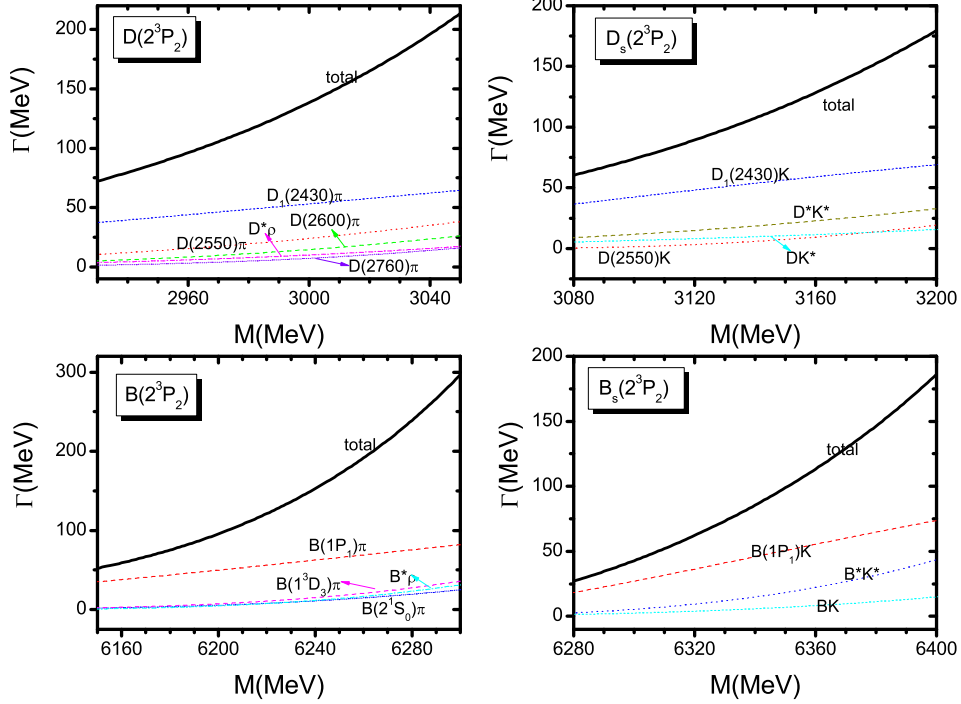


FIG. 11: The partial decay width and total decay widths of the 2^3P_2 states in the heavy-light mesons as functions of the mass. In the figure, we have hidden some decay channels because of their small partial decay widths. The mass of the $B(1^3D_3)$ is adopted as $M = 5970$ MeV.

G. The 2^3P_2 states

In the D -meson family, the predicted mass of the 2^3P_2 is about 3.02 GeV [33, 39, 40]. To know about the decay properties of the $D(2^3P_2)$, in Fig. 11 we plot its decay width as a function of mass in the range of $M = (2.95 \sim 3.05)$ GeV. From the figure, it is found that the $D(2^3P_2)$ mainly decays into the $D_1(2430)\pi$, $D(2550)\pi$ and $D(2600)\pi$ channels. The predicted total decay width of $D(2^3P_2)$ is $\Gamma \simeq 150$ MeV, if we adopt the mass ~ 3.01 GeV as predicted in Ref. [40].

Considering the newly observed state $D_J^*(3000)$ as a candidate of $D(2^3P_2)$, we find that the observed mass, width ($\Gamma \simeq 110$ MeV) and natural parity of the $D_J^*(3000)$ [2] can be well explained. However, the $D\pi$ decay channel is not the

main decay channel of $D(2^3P_2)$, which is tiny compared with the dominant decay mode $D_1(2430)\pi$. The predicted branching fraction is

$$\frac{\Gamma[D\pi]}{\Gamma_{\text{total}}} \simeq 1 \sim 2\%, \quad (21)$$

which is compatible with the predictions in Refs. [10, 34]. To clarify the nature of the $D_J^*(3000)$, it is suggested to further observe this state in the $D_1(2430)\pi$ channel. If the $D_J^*(3000)$ corresponds to the $D(2^3P_2)$ state, it should be found in the $D_1(2430)\pi$ channel as well.

In the D_s -meson family, the predicted mass of the 2^3P_2 is about 3.15 GeV [33, 39, 40]. To know about the decay properties of the $D_s(2^3P_2)$, in Fig. 11 we plot its decay width as a function of mass in the range of $M = (3.1 \sim 3.2)$ GeV. From

the figure, it is found that the $D_s(2^3P_2)$ mainly decays into the $D_1(2430)K$ channel. The total decay width of $D_s(2^3P_2)$ is $\Gamma \simeq 120 \pm 60$ MeV. This resonance might be observed in the $D_1(2430)K$ final states.

In the B -meson family, the predicted mass of the 2^3P_2 is about $6.19 \sim 6.26$ GeV [39, 40]. In Fig. 11, we have plotted its decay width as a function of mass in the range of $M = (6.15 \sim 6.3)$ GeV. From the figure it is found that the total decay width of $B(2^3P_2)$ is $\Gamma \simeq (50 \sim 300)$ MeV. The $B(2^3P_2)$ mainly decays into the $B_1(5721)\pi$ channel.

In the B_s -meson family, the predicted mass of the 2^3P_2 is about $6.29 \sim 6.36$ GeV [33, 39, 40]. The decay properties of the $B(2^3P_2)$ have been shown in Fig. 11. From the figure, it is found that the $B_s(2^3P_2)$ mainly decays into $B_1(5721)K$. The total decay width of $B_s(2^3P_2)$ is $\Gamma \simeq (30 \sim 180)$ MeV. This state might be observed in the $B_1(5721)K$ channel.

In a brief, the decay widths of the 2^3P_2 excitations in the heavy-light spectroscopy are not very broad, they have good potentials to be observed in their dominant decay channels.

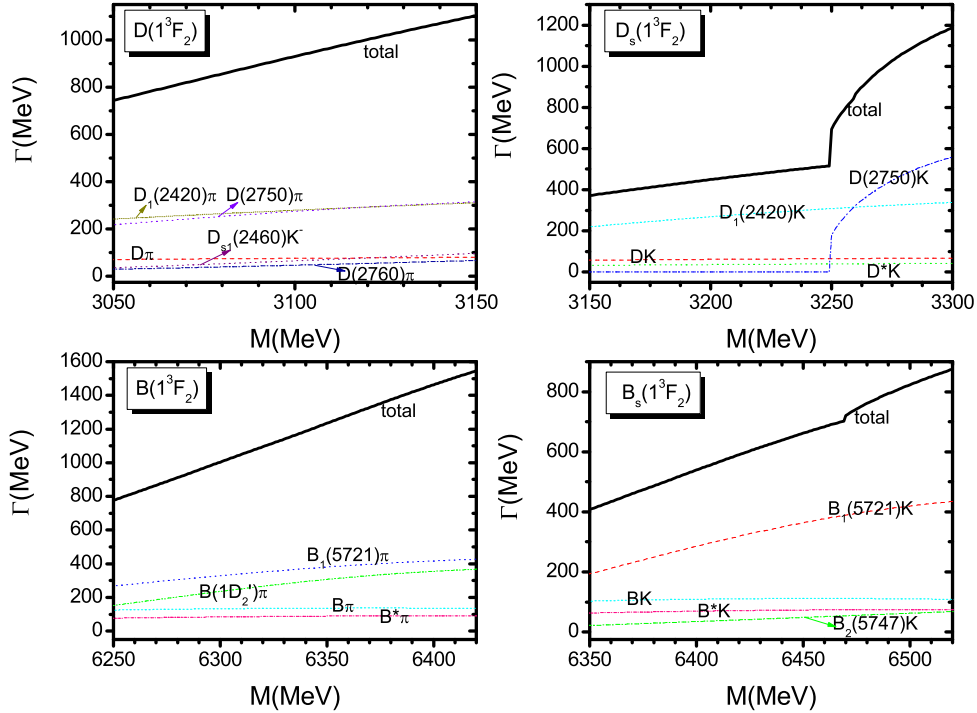


FIG. 12: The partial decay width and total decay widths of the 1^3F_2 states in the heavy-light mesons as functions of the mass. In the figure, we have hidden some decay channels because of their small partial decay widths. The mass of the $B(1D_2')$ is adopted as $M = 6025$ MeV.

H. The 1^3F_2 states

In the D -meson family, the predicted mass for the 1^3F_2 state is ~ 3.1 GeV [33, 39, 40]. Considering the uncertainties of the mass, in Fig. 12 we plot the partial decay widths and total width as functions of mass in the range of $M = (3.05 \sim 3.15)$ GeV. It is found that the 1^3F_2 state in the D -meson family is a very broad state with a width of $\Gamma \simeq 900 \pm 200$ MeV. This state dominantly decays into the $D(2420)\pi$, $D(2750)\pi$ and $D\pi$ channels. The partial widths for the $D_s(2460)\pi$, $D^*\pi$ and $D(2760)\pi$ are also sizable.

In the D_s -meson family, the predicted mass for the 1^3F_2 state is ~ 3.23 GeV [33, 39, 40]. Considering the uncertainties of the mass, we vary the mass in the range of $M = (3.15 \sim 3.30)$ GeV, the strong decay properties of the 1^3F_2 state have been shown in Fig. 12 as well. From the figure it is seen that the decay width of the $D_s(1^3F_2)$ state is $\Gamma \sim 500$ MeV. The main decay channels are $D(2420)K$, DK and D^*K . If the $D(2750)K$ channel is opened, the $D(2750)K$ mode will dominate the decays, the decay width of the $D_s(1^3F_2)$ state will become very broad with a width of $\Gamma > 700$ MeV.

In the B -meson family, the predicted mass for the 1^3F_2 state

is $6.26 \sim 6.41$ GeV [33, 39, 40]. In this mass region, we show the strong decay properties of the $B(1^3F_2)$ state in Fig. 12. It is found that this state is also a very broad state with a width of $\Gamma \simeq 1200 \pm 400$ MeV. It mainly decays into the $B(1P_1)\pi$, $B(1D_2)\pi$, $B\rho$ and $B^*\pi$ channels.

In the B_s -meson family, the predicted mass for the 1^3F_2 state is $6.37 \sim 6.50$ GeV [39, 40]. In this mass region, we study the strong decays of the $B_s(1^3F_2)$ state, our results are shown in Fig. 12 as well. It is seen that this state is a very

broad state with a width of $\Gamma \simeq 400 \sim 900$ MeV. The strong decays are governed by the $B(1P_1)K$. The other main decay channels are BK , B^*K and $B(1^3P_2)K$.

As a whole, the 1^3F_2 excitations in the heavy-light mesons are very broad resonances, whose width is larger than 400 MeV. Such broad states might be difficult to observe in experiments, which might explain why these states are still missing in the heavy-light meson spectroscopy.

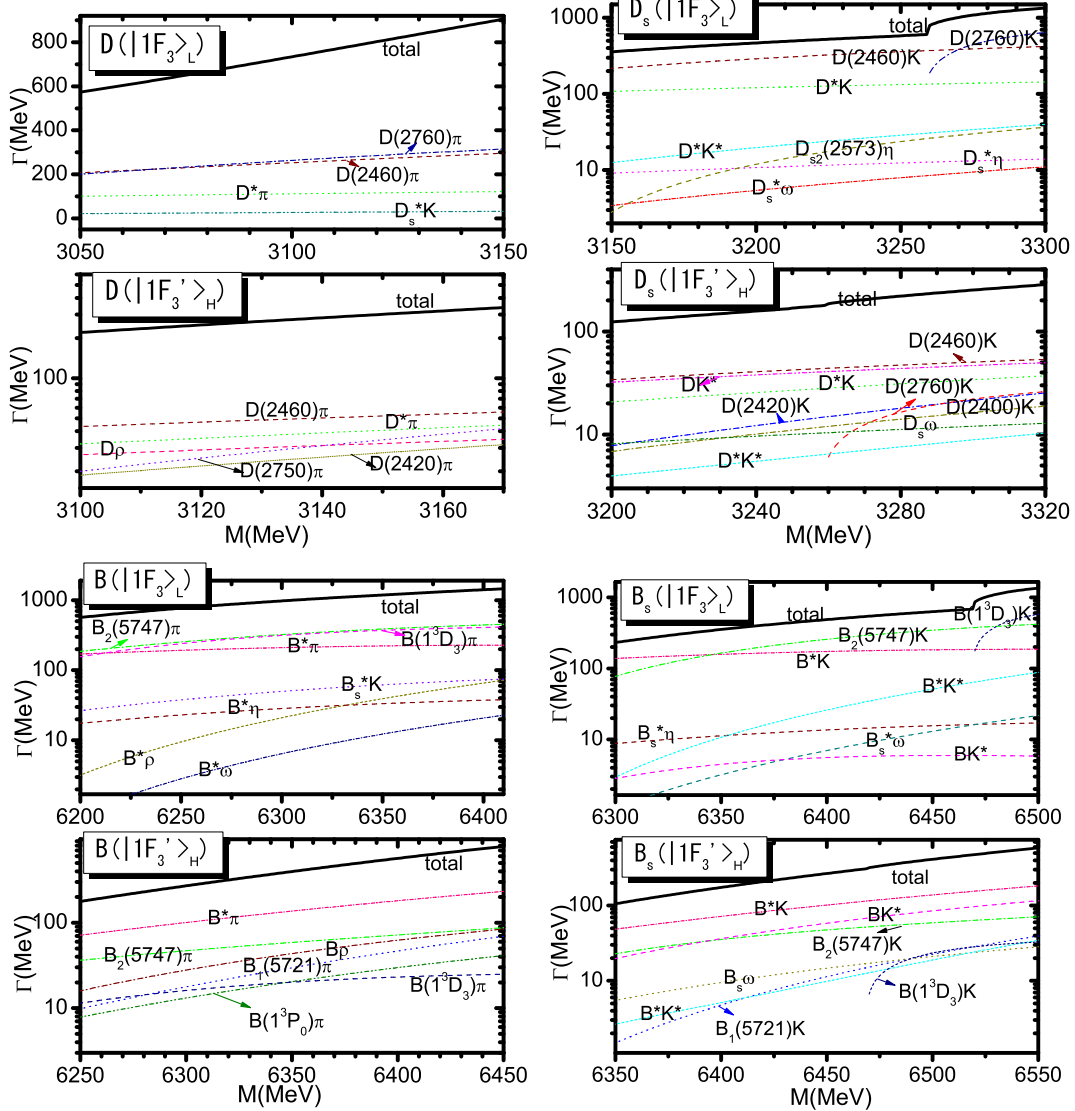


FIG. 13: The partial decay width and total decay width for the mixed states via the 1^1F_3 - 1^3F_3 mixing as functions of the mass. The mixing angle is adopted as $\phi_{1F} = -49.1^\circ$. In the figure, we have hidden some decay channels because of their small partial decay widths.

I. The 1^3F_3 - 1^3F_3 mixing

Since the heavy-light mesons are not charge conjugation eigenstates, state mixing between spin $S = 0$ and $S = 1$

states with the same J^P can occur via the spin-orbit interac-

tions [27, 29, 45]. The physical states with $J^P = 3^+$ can then be described as

$$\begin{pmatrix} |1F_3\rangle_L \\ |1F_3'\rangle_H \end{pmatrix} = \begin{pmatrix} \cos\phi_{1F} & \sin\phi_{1F} \\ -\sin\phi_{1F} & \cos\phi_{1F} \end{pmatrix} \begin{pmatrix} |1^1F_3\rangle \\ |1^3F_3\rangle \end{pmatrix}, \quad (22)$$

where the subscriptions L and H stand for the low mass and high mass of the physical states after the mixing. In the heavy quark symmetry limit, the mixing angle ϕ_{1F} is predicted to be $\phi_{1F} \simeq -49.1^\circ$ [40]. It should be mentioned that with the mixing angle $\phi_{1F} \simeq -49.1^\circ$, the low-mass state $|1F_3\rangle_L$ usually has a broad width, while the high-mass state $|1F_3'\rangle_H$ has a narrower width.

In the D -meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3'\rangle_H$ are about 3.07 ~ 3.13 GeV and 3.12 ~ 3.15 GeV, respectively [39, 40]. With the mixing angle $\phi_{1F} \simeq -49.1^\circ$ derived in the heavy quark symmetry limit, we predicted the strong decay properties of the mixed states $D(|1F_3\rangle_L)$ and $D(|1F_3'\rangle_H)$ in their possible mass region. The results are shown in Fig. 13. It is found that the $D(|1F_3\rangle_L)$ is

a very broad state with a width of $\Gamma \simeq 600 \sim 900$ MeV. Its strong decays are dominated by the $D(2760)\pi$, $D(2460)\pi$ and $D^*\pi$ channels. The width of the $D(|1F_3'\rangle_H)$ is $\Gamma \simeq 200 \sim 300$ MeV, which is much narrower than that of the low-mass state. The $D(|1F_3'\rangle_H)$ mainly decays into the $D(2460)\pi$, $D^*\pi$, $D\rho$, $D(2420)\pi$ and $D(2750)\pi$ channels.

In the D_s -meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3'\rangle_H$ are about 3.20 ~ 3.26 GeV and 3.25 ~ 3.27 GeV, respectively [39, 40]. The strong decay properties of these mixed states are also studied in their possible mass region. Our predictions are shown in Fig. 13. It is seen that the width of the low-mass mixed state $D_s(|1F_3\rangle_L)$ is very broad, which is in the range of $\Gamma \simeq 400 \sim 800$ MeV. Its strong decays are governed by the $D(2460)K$ and D^*K channels. The high-mass state $D_s(|1F_3'\rangle_H)$ has a much narrower decay width, i.e., $\Gamma \simeq 100 \sim 250$ MeV. This high-mass state dominantly decays into the $D(2460)K$, D^*K , and DK^* channels.

In the B -meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3'\rangle_H$ are about 6.22 ~ 6.39 GeV and 6.27 ~ 6.42 GeV, respectively [39, 40]. In the possible mass region, we study the strong decay properties of these mixed states in the B -meson family. The results are shown in Fig. 13. It is found that the low-mass mixed state $B(|1F_3\rangle_L)$ is a very broad state with a width of $\Gamma \simeq 650 \sim 1400$ MeV. This state dominantly decays into the $B_2(5747)\pi$, $B(1^3D_3)\pi$, and $B^*\pi$ channels. While for the $B(|1F_3'\rangle_H)$, the decay width is $\Gamma \simeq 200 \sim 650$ MeV, which is sensitive to the mass. If the $B(|1F_3'\rangle_H)$ state has a smaller mass as predicted in Ref. [39], it might be observed in its main decay channels $B^*\pi$ and $B_2(5747)\pi$.

In the B_s -meson family, the predicted masses for the mixed states $|1F_3\rangle_L$ and $|1F_3'\rangle_H$ are about 6.33 ~ 6.47 GeV and 6.38 ~ 6.52 GeV, respectively [39, 40]. Considering the uncertainties of the mass, we have plotted the strong decay properties of these mixed states as functions of the mass in Fig. 13. From the figure, it is seen that the $|1F_3\rangle_L$ state in the B_s -meson family has a width of $\Gamma \simeq 200 \sim 700$ MeV, whose strong decays are dominated by the $B_2(5747)K$ and B^*K channels. The high-mass state $|1F_3'\rangle_H$ has a relatively smaller width, which is $\Gamma \simeq 100 \sim 500$ MeV. Its strong decays are governed by

the B^*K , BK^* , and $B_2(5747)K$ channels. These mixed states $|1F_3\rangle_L$ and $|1F_3'\rangle_H$ in the B_s -meson family might be observed in the B^*K channel, if they have a smaller mass as predicted in Ref. [39].

In the calculations we have adopted the ideal mixing angle $\phi_{1F} \simeq -49.1^\circ$ extracted from the heavy quark symmetry limit. This ideal mixing angle might have some uncertainties. To see the effects of the uncertainties of the mixing angle on the strong decay properties, we have plotted the partial decay widths and total decay width as functions of the mixing angle in Fig. 14. From the figure, it is seen that strong decay properties of the mixed states do not change obviously, if we consider an uncertainty $\pm 20^\circ$ around the ideal mixing angle $\phi_{1F} \simeq -49.1^\circ$.

In summary, the mixed states $|1F_3\rangle_L$ in the heavy-light mesons are usually broad states, they should be difficult to find in experiments. The decay width of the high-mass mixed states $|1F_3'\rangle_H$ is relatively narrower than that of the low-mass states. These high-mass states $|1F_3'\rangle_H$ might be observed in future experiments if they have a smaller mass as predicted in Ref. [39]. However, these high-mass states $|1F_3'\rangle_H$ might be very broad states if their masses are as large as those predicted in Ref. [40].

J. The 1^3F_4 states

In the D -meson family, the predicted mass for the 1^3F_4 state is 3.09 ~ 3.19 GeV [33, 39, 40]. In this mass region, the

predicted strong properties are shown in Fig. 15. The width of the $D(1^3F_4)$ state is $\Gamma \simeq 230 \pm 70$ MeV. It dominantly decays into the $D^*\rho$, $D\pi$, $D(2430)\pi$, $D(2460)\pi$, $D^*\pi$ and $D(2760)\pi$

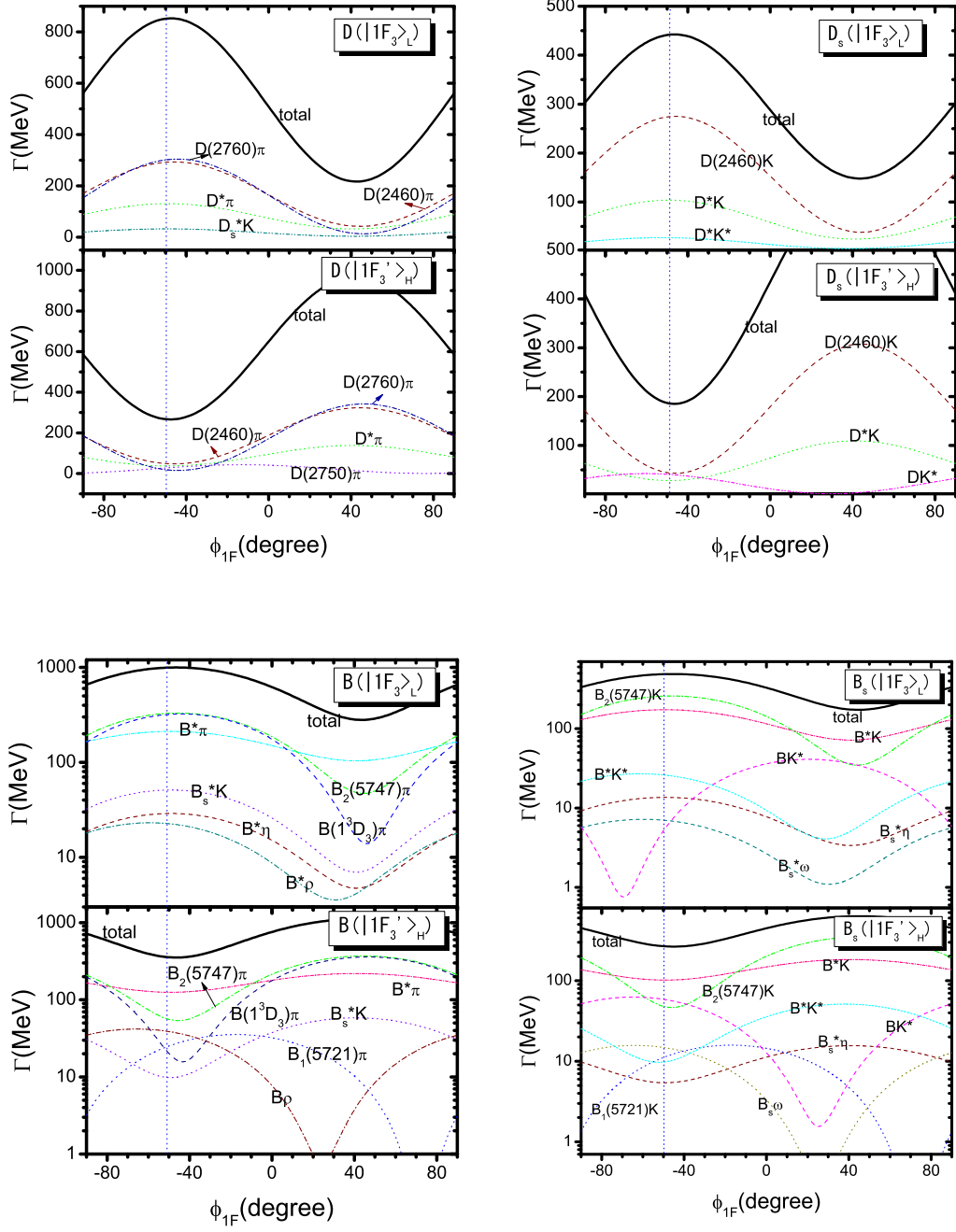


FIG. 14: The partial decay width and total decay width for the mixed states via the 1^1F_3 - 1^3F_3 mixing as functions of the mixing angle. The masses for the low-mass mixed states $D(|1F_3\rangle_L)$, $D_s(|1F_3\rangle_L)$, $B(|1F_3\rangle_L)$ and $B_s(|1F_3\rangle_L)$ are adopted as 3100, 3230, 6305, and 6400 MeV, respectively. The masses for the high-mass mixed states $D(|1F_3'\rangle_H)$, $D_s(|1F_3'\rangle_H)$, $B(|1F_3'\rangle_H)$ and $B_s(|1F_3'\rangle_H)$ are adopted as 3130, 3260, 6335, and 6450 MeV, respectively. In the figure, we have hidden some decay channels for their small partial decay widths.

channels. The branching ratios for $D^*\pi$ and $D\pi$ are 10% and 11%, respectively.

We find that if taking the newly observed natural parity resonance $D_J^*(3000)$ as a candidate of the $D(1^3F_4)$, the strong decay properties of $D_J^*(3000)$ could be well explained (see

Tab. IV). However, the predicted mass is about 100 ~ 200 MeV larger than the data. If the $D_J^*(3000)$ corresponds to the $D(1^3F_4)$ state indeed, it should be observed in both $D\pi$ and $D^*\pi$ channels. To clarify the puzzles in the $D_J^*(3000)$, more observations are needed.

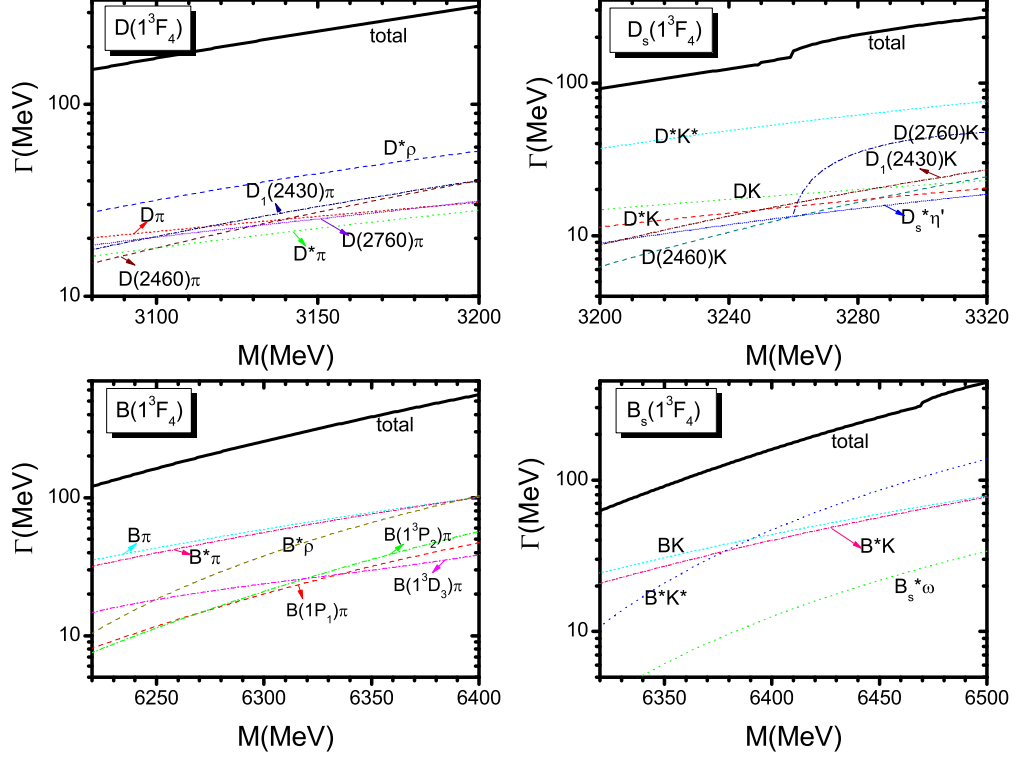


FIG. 15: The partial decay width and total decay width for the 1^3F_4 excitations as functions of the mass. In the figure, we have hidden some decay channels because of their small partial decay widths.

TABLE IV: The strong partial decay widths (MeV) and total width (MeV) of $D^*(3000)$ as a candidate of $D(1^3F_4)$.

Mode	$D\pi$	$D^*\pi$	$D(2420)\pi$	$D(2430)\pi$	$D(2460)\pi$	$D(2550)\pi$	$D(2600)\pi$	$D(2750)\pi$	$D(2760)\pi$
Width	14.9	11.0	2.3	9.2	7.0	0.9	5.9×10^{-2}	4.6	11.4
Mode	$D\eta$	$D\eta'$	$D^*\eta$	$D^*\eta'$	$D(2420)\eta$	$D(2430)\eta$	$D(2460)\eta$	$D_s(2460)K$	D_s^*K
Width	1.0	3.1×10^{-2}	0.5	8.9×10^{-5}	5.6×10^{-4}	1.0×10^{-3}	8.1×10^{-12}	3.6×10^{-3}	0.8
Mode	D_sK	$D\omega$	$D^*\omega$	$D\rho$	$D^*\rho$	D_sK^*	$D_s^*K^*$	Total	
Width	2.1	9.2×10^{-6}	4.5	1.6	14.5	5.9×10^{-7}	6.4×10^{-4}	86.4	

In the D_s -meson family, the predicted mass for the 1^3F_4 state is $3.22 \sim 3.30$ GeV [33, 39, 40]. The strong decay properties are shown in Fig. 15 as well. In the possible mass region, the predicted width of the $D_s(1^3F_4)$ state is $\Gamma \simeq 200 \pm 100$ MeV. The main decay channels are D^*K^* , DK , D^*K , $D(2430)K$, and $D(2460)K$. If the mass of the $D_s(1^3F_4)$ state is larger than the threshold of $D(2760)K$, the $D(2760)K$ channel will become a dominant decay channel. The predicted partial width ratio of DK and D^*K is

$$\frac{\Gamma[DK]}{\Gamma[D^*K]} \simeq 1.2. \quad (23)$$

This state might be observed in both DK and D^*K channels.

In the B -meson family, the predicted mass for the 1^3F_4 state is $6.22 \sim 6.38$ GeV [33, 39, 40]. In this mass region, we study the strong decay properties of the $B(1^3F_4)$ state, which are shown in Fig. 15. From the figure, it is seen that the strong decays of the $B(1^3F_4)$ state are dominated by the $B\pi$ and $B^*\pi$, their partial width ratio is

$$\frac{\Gamma[B\pi]}{\Gamma[B^*\pi]} \simeq 1.1, \quad (24)$$

which is less sensitive to the mass. Furthermore, the other decay channels $B^*\rho$, $B(1^3D_3)\pi$, $B(1^2P_3)\pi$ and $B(1P_1)\pi$ also have

obvious contributions to the strong decays. The decay width is sensitive to the mass. In the possible mass region, the predicted width of the $B(1^3F_4)$ state is $\Gamma \simeq 120 \sim 480$ MeV. If this state has a mass of ~ 6.2 GeV as predicted in Ref. [39], it might be observed in future experiments. However, if the mass is close to ~ 6.4 GeV as predicted in Ref. [40], the $B(1^3F_4)$ state might be difficult to be found for its broad width.

In the B_s -meson family, the predicted mass for the 1^3F_4 state is $6.33 \sim 6.48$ GeV [33, 39, 40]. In this mass region, we have shown the strong decay properties in Fig. 15. The decay width of the $B_s(1^3F_4)$ is sensitive to the mass. The total decay width is $\Gamma \simeq 60 \sim 400$ MeV, which bares a large uncertainty. Its strong decays are dominated by the BK , B^*K and B^*K^* channels. The predicted partial width ratio of the BK and B^*K is

$$\frac{\Gamma[BK]}{\Gamma[B^*K]} \simeq 1.2, \quad (25)$$

which is insensitive to the mass. If this state has a smaller mass of ~ 6.34 GeV as predicted in Ref. [39], it should be a narrow state with a width of $\Gamma \simeq 80$ MeV. In this case, the $B_s(1^3F_4)$ might be observed in both BK and B^*K channels.

As a whole the decay widths of the 1^3F_4 excitations in the heavy-light mesons are sensitive to the mass. If these states have a smaller mass as predicted in Ref. [39], they might have relatively narrow width $\Gamma \sim 100$ MeV. In this case, these excitations have good discovery potentials in future experiments. However, if these states have a larger mass as predicted in Ref. [40], they might be difficult to be found in experiments for their broad widths. Finally, it should be pointed out that the newly observed natural parity resonance $D_J^*(3000)$ seems to be a good candidate of the $D(1^3F_4)$, although its mass is about $100 \sim 200$ MeV less than the model predictions.

IV. SUMMARY

In the chiral quark model framework, we systematically study the strong decays of the higher excited heavy-light mesons from the first radially excited states up to the first F -wave states. We summarize our major results as follows.

The first radially excited states 2^1S_0 in the D -, D_s -, B - and B_s -meson spectroscopy are narrow states, and their predicted width might be less than 100 MeV. If the $D(2550)$ corresponds to this excitation indeed, the other radially excited states $D_s(2^1S_0)$, $B(2^1S_0)$ and $B_s(2^1S_0)$ are most likely to be also observed in the D^*K , $B^*\pi$ and B^*K final states, respectively, for their narrower widths.

Configuration mixing might exist between the low-lying $J^P = 1^-$ states 2^3S_1 and 1^3D_1 . Both $D(2600)$ and $D_{s1}(2700)$ could be assigned as the low-mass mixed state $| (SD)_1 \rangle_L$ via the 2^3S_1 - 1^3D_1 mixing. As flavor partners of the $D(2600)$ and $D_{s1}(2700)$, the low-mass mixed states in the B - and B_s -meson families, $B(|(SD)_1 \rangle_L)$ and $B_s(|(SD)_1 \rangle_L)$, should have a relatively narrow width. They are most likely to be observed in future experiments. In the high-mass mixed states, the widths of the charmed-strange state $D_s(|(SD)_1' \rangle_H)$ and the bottom-strange state $B_s(|(SD)_1' \rangle_H)$ are as comparable as the low-mass

mixed states, which might be observed in the DK and BK channels, respectively. While, the bottom state $B(|(SD)_1' \rangle_H)$ is a broad state, which might be difficult to be found in experiments. It should be pointed out that the newly observed resonance $D_{s1}(2860)$ by LHCb Collaboration might correspond to the physical partners of the $D_{s1}(2700)$, i.e., the high-mass state $D_s(|(SD)_1' \rangle_H)$.

In the low-lying D -wave states with $J^P = 2^-$, 1^1D_2 and 1^3D_2 , there might be configuration mixing as well. The narrow resonances $D(2750)$ and $D_{sJ_2}(2860)$ might be classified as the high-mass mixed state $|1D_2' \rangle_H$ via the 1^1D_2 - 1^3D_2 mixing. The other high-mass mixed states in the B - and B_s -meson families, $B(|1D_2' \rangle_H)$ and $B_s(|1D_2' \rangle_H)$ have a narrow width as well. These two narrow states might be observed in the $B^*\pi$ and B^*K , respectively. However, the low-mass mixed state $|1D_2 \rangle_L$ should be a broad state. The typical total decay widths for the mixed states $D(|1D_2 \rangle_L)$ and $B(|1D_2 \rangle_L)$ in the D - and B -meson families are usually larger than 300 MeV, which might be too broad to be observed in experiments. While the widths for the mixed states $D_s(|1D_2 \rangle_L)$ and $B_s(|1D_2 \rangle_L)$ containing a strange quark are ~ 250 MeV, which still have some possibilities to be observed in future experiments.

For the low-lying D -wave excitations with $J^P = 3^-$, 1^3D_3 , in the D -, D_s -, B - and B_s -meson spectroscopy, we might have observed three excitations $D(2760)$, $D_{sJ_3}(2860)$ and $B(5970)$ in recent experiments. The last unobserved one in the B_s -meson family should be a narrow state with a width of $\Gamma \simeq (25 \sim 75)$ MeV, which is most likely to be found in the BK and B^*K channels.

No evidence of the second P -wave excitations with $J^P = 0^+$ is found in experiments. In the D - and B -meson families, the $D(2^3P_0)$ and $B(2^3P_0)$ excitations should be very broad states, whose widths are larger than 400 MeV. It might be difficult to be found in experiments. However, the widths of the resonances $D_s(2^3P_0)$ and $B_s(2^3P_0)$ in the D_s - and B_s -meson spectroscopy are relatively narrower, which might be observed in the DK and BK channels, respectively.

The physical states of the second P -wave states with $J^P = 1^+$ might be mixed states between the 2^1P_1 and 2^3P_1 . The strong decay properties of both $D_{sJ}(3040)$ and $D_J(3000)$ can be explained by assigning them as the low-mass mixed state $|2P_1 \rangle_L$ via the 2^1P_1 - 2^3P_1 mixing with a mixing angle in the range of $\phi_{2P} = -(20 \sim 26)^\circ$. The mixed state $B_s(|2P_1 \rangle_L)$ in the B_s -meson family is the narrowest state in these low-mass mixed states, thus, it is most likely to be observed in the B^*K channel. On the contrary, the mixed state $B(|2P_1 \rangle_L)$ in the B -meson family might have the broadest width in these low-mass mixed states, thus, its discovery potentials might be small. The high-mass mixed states $D(|2P_1' \rangle_H)$, $D_s(|2P_1' \rangle_H)$, $B(|2P_1' \rangle_H)$ and $B_s(|2P_1' \rangle_H)$ are usually narrower than those of low-mass states. These states dominantly decay into the first P -wave states by emitting a light pseudoscalar meson. However, experimental analysis of these decay channels is still absent, which might explain why these broader mixed states $D_{sJ}(3040)$ and $D_J(3000)$ have been first found in experiments. It is strongly suggested to carry out experimental analysis of the final states containing a low-lying P -wave state with $J^P = 0^+, 1^+$.

No evidence of the second P -wave excitations with $J^P = 2^+$ in the D -, D_s -, B - and B_s -meson spectroscopy is found in experiments. Our calculations indicate that these P -wave excitations $D(2^3P_2)$, $D_s(2^3P_2)$, $B(2^3P_2)$ and $B_s(2^3P_2)$ have a relatively narrow width, their strong decays are governed by the $D_1(2430)\pi$, $D_1(2430)K$, $B(1P_1)\pi$ and $B(1P_1)K$, respectively. Thus, it might be a good chance for us to find the $D(2^3P_2)$ and $D_s(2^3P_2)$ excitations by analyzing the data in the $D_1(2430)\pi$ and $D_1(2430)K$ final states, respectively.

The 1^3F_2 excitations in the heavy-light mesons are most likely to be very broad resonances, whose widths are larger than 400 MeV. Such broad states might be difficult to be observed in experiments, which might explain why these states are still missing in the heavy-light meson spectroscopy.

Considering configuration mixing in the first F -wave states with $J^P = 3^+$, we predict that the low-mass mixed states $|1F_3\rangle_L$ in the heavy-light mesons are usually broad states, they should be difficult to be found in experiments. While the decay widths of the high-mass mixed states $|1F_3'\rangle_H$ are relatively narrower than those of the low-mass states. These high-mass states $|1F_3'\rangle_H$ might be observed in future experiments if they have a smaller mass as predicted in Ref. [39]. The optimal observed channels for the $D(|1F_3'\rangle_H)$, $D_s(|1F_3'\rangle_H)$, $B(|1F_3'\rangle_H)$ and $B_s(|1F_3'\rangle_H)$ are $D^*\pi$, D^*K , $B^*\pi$ and B^*K , respectively. However, these high-mass states $|1F_3'\rangle_H$ might be very broad states if their mass is as large as that predicted in Ref. [40].

For the 1^3F_4 excitations, their decay widths are sensitive to the mass. If these states have a smaller mass as predicted in Ref. [39], they might have relatively narrow width $\Gamma \sim 100$ MeV. In this case, these excitations have good observation potentials in future experiments. The optimal observed channels

for the $D(1^3F_4)$, $D_s(1^3F_4)$, $B(1^3F_4)$ and $B_s(1^3F_4)$ are $D\pi/D^*\pi$, DK/D^*K , $B\pi/B^*\pi$ and BK/B^*K , respectively. However, if these states have a larger mass as predicted in Ref. [40], they might be difficult to be found in experiments for their broad widths. Taking the newly observed natural parity resonance $D_J^*(3000)$ as a candidate of the $D(1^3F_4)$, we find that the theoretical predictions of the strong decay properties are consistent with the observation. However, the predicted mass is inconsistent with the observation, which is about 100 ~ 200 MeV larger than the data. If the $D_J^*(3000)$ corresponds to the $D(1^3F_4)$ state indeed, it should be observed in both $D\pi$ and $D^*\pi$ channels. To clarify the puzzles in the $D_J^*(3000)$, more observations are needed.

Finally, it should be pointed out that the chiral quark model still has some limitations. For example, the simple harmonic oscillator wave functions of the resonances have been adopted in the calculations, which should be different from the realistic wave functions more or less. Thus, the wave functions might bring some uncertainties to our predictions. Furthermore, our model is a nonrelativistic model, the relativistic effects could bring some uncertainties to the decay widths as well. Thus, we might only give a qualitative prediction of the strong decay properties for some resonances.

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